Graduation Numeracy Assessment
Sample Assessment Student-Choice Component: Scoring Guide and Student Exemplars
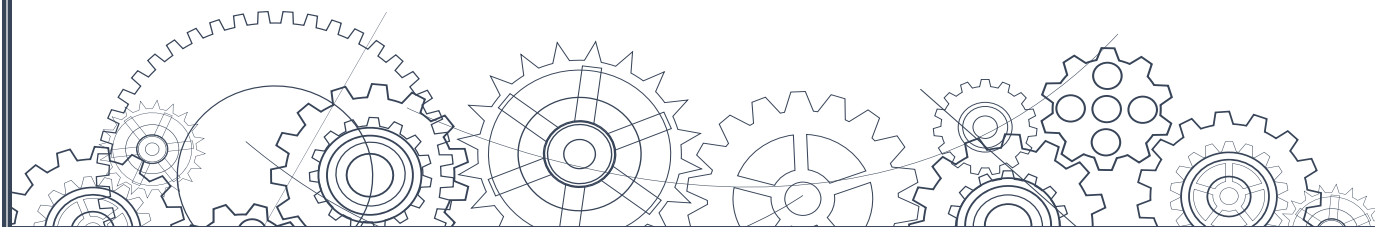
BRITISH COLUMBIA | Ministry of Education

2017

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Numeracy Assessment Rubric

1	2	3	4
The student demonstrates an inadequate understanding of the situation. The strategy is ineffective. The solution may contain fundamental mathematical errors. The reasoning is missing or irrelevant; the logic does not reference the problem.	The student demonstrates a basic understanding of the situation. The strategy is unclear and/or incomplete. The solution may contain mathematical errors. The reasoning is unclear; but the logic correctly references some aspects of the problem.	The student demonstrates an adequate understanding of the situation. The strategy is sensible but has some inconsistencies. The solution may contain minor mathematical errors. The reasoning is evident, and the logic references most aspects of the problem.	The student demonstrates a proficient understanding of the situation. The strategy is effective and comprehensive. The solution may contain minor mathematical errors that do not affect the demonstration of proficiency. The reasoning is clear and the logic references all aspects of the problem.

Snapshot

NR	No response (answer page is blank).	0	Information simply recopied from the problem. Diagrams or calculations are unrelated to the problem. Response does not address the purpose of the task. An incorrect mathematical solution with no work shown. Inappropriate response (contains profanity, inappropriate diagram or language). All work is erased or crossed out. Any zero score must include rationale and be approved by the section head.

Note:

All rationales correspond to the revised rubric snapshot shown above. The rubric is used for holistic scoring of all extensive constructed response questions. For each exemplar in the document, the descriptors identified from the rubric are the best representation for that exemplar. The bullet points underneath describe, in whole or in part, the evidence found within the student work, but do not form a complete list of what is needed for that particular score, and therefore should not be used as such.

Elaborations

|	1	2	3	4
Interpret	• Demonstrates limited reasoning skills in interpreting the situation. • Makes fundamental errors when making decisions about relevant information given in text, symbols, or graphical forms.	• Demonstrates basic reasoning skills in interpreting the situation. • Makes some errors when making decisions about relevant information given in text, symbols, or graphical forms.	• Demonstrates effective reasoning skills in interpreting the situation. • Makes minor errors when making decisions about relevant information given in text, symbols, or graphical forms.	• Demonstrates advanced reasoning skills in interpreting the situation. • Makes no errors when making decisions about relevant information given in text, symbols, or graphical forms.
Apply	• Demonstrates limited success in selecting and applying an effective approach for the situation. • Creates algebraic expressions or equations with fundamental errors in logic.	• Demonstrates some success in selecting and applying an effective approach for the situation. • Creates algebraic expressions or equations with some errors in logic.	• Demonstrates success in selecting and applying an effective approach for the situation. • Creates algebraic expressions or equations with minor errors in logic.	• Demonstrates success in selecting and applying a sophisticated approach for the situation. • Creates algebraic expressions or equations with no errors in logic.
Solve	• Demonstrates limited use of mathematical concepts and skills involved in the situation. • Mathematical solution is incomplete or incorrect, with fundamental calculation errors.	• Demonstrates basic use of mathematical concepts and skills involved in the situation. • Mathematical solution is incorrect, with some calculation errors.	• Demonstrates effective use of mathematical concepts and skills involved in the situation. • Mathematical solution is incorrect, with minor calculation errors.	• Demonstrates advanced use of mathematical concepts and skills involved in the situation. • Mathematical solution is correct, with no calculation errors.
Analyze	• No attempt to assess the reasonableness of solution, or attempt contains fundamental errors in reasoning. • Demonstrates limited ability to suggest possible limitations of a solution, improvements to an approach, or alternate situations to which solution can be applied.	• Assesses the reasonableness of solution, but contains some errors in reasoning. • Demonstrates basic ability to suggest possible limitations of a solution, improvements to an approach, or alternate situations to which solution can be applied.	• Effective analysis about the reasonableness of solution, but may contain minor errors in reasoning. • Demonstrates effective ability to suggest possible limitations of a solution, improvements to an approach, or alternate situations to which solution can be applied.	• Insightful analysis about the reasonableness of solution. • Demonstrates advanced ability to suggest possible limitations of a solution, improvements to an approach, or alternate situations to which solution can be applied.
Communicate	• Demonstrates limited ability to communicate using mathematical language (e.g., graphs, symbols). • Presents limited or irrelevant evidence for the solution.	• Demonstrates basic ability to communicate using mathematical language (e.g., graphs, symbols). • Presents some evidence for the solution; solution may have inconsistencies or be difficult to follow.	• Demonstrates effective ability to communicate using mathematical language (e.g., graphs, symbols). • Presents effective evidence for the solution using logical arguments.	• Demonstrates advanced ability to communicate using mathematical language (e.g., graphs, symbols). • Presents sophisticated evidence for the solution using logical arguments.

Rubric and elaborations will be further refined following the first assessment administration and will be available in the spring of 2018.

A newspaper headline grabs your attention.

DAILY NEWS

Water use skyrockets

Recent studies have predicted water shortages.

Average Water Use (L/person/day)
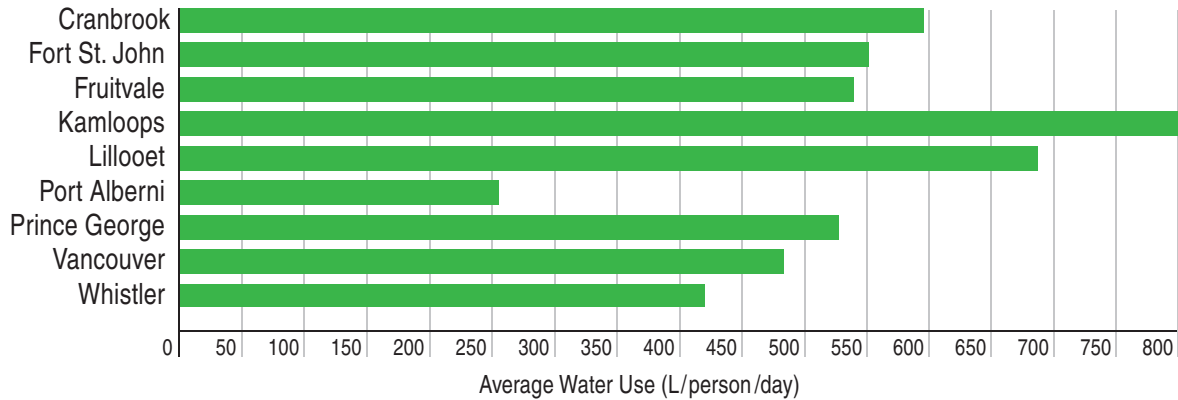
Indoor Water Use with Conventional Appliances and Fixtures

Toilet	13 L/flush		Tap	8 L/minute
Shower	10 L/minute		Dishwasher	40 L/cycle
Bath	100 L/bath		Leaks	3 L/day
Clothes Washer	150 L/cycle			

$2.75 Our office 14 1st Street   Volume 51 / Est. 1987

High-efficiency appliances and fixtures can help reduce the amount of water we use.

Indoor Water Use with High-Efficiency Appliances and Fixtures

Toilet
6 L/flush

Shower
8 L/minute

Dishwasher
16 L/cycle

Tap
6 L/minute

Bath
50 L/bath

Clothes Washer
60 L/cycle
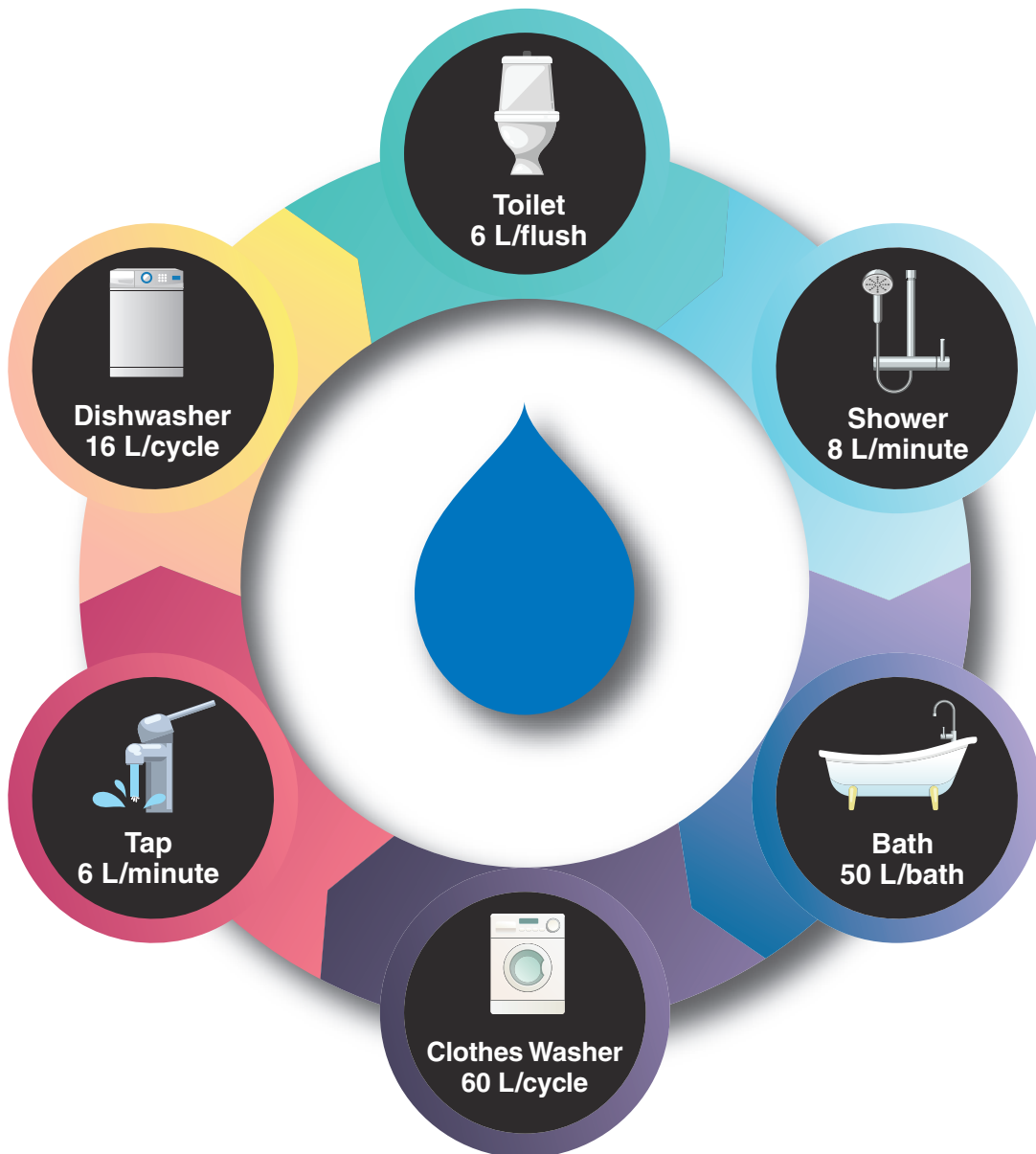
13. You want to reduce your personal water use to 1050 L/week. You install high-efficiency appliances and fixtures, and change your water-use habits.

    Plan a water budget for yourself for 1 week that meets this goal using the high-efficiency appliances and fixtures.

    Explain and justify your solution.

    You must use everything in the table below at least once in the week.

High-Efficiency Appliances and Fixtures
Shower and/or bath
Toilet
Tap
Dishwasher
Clothes Washer

    This question is to be answered on paper.

Summary of Requirements for a Level 4:

▶ Use less than 1050 L/week

▶ Use each of the items in the table (shower or bath, toilet, tap, dishwasher, clothes washer) at least once

▶ Be realistic (e.g., cannot flush toilet once per week)

▶ Can do a daily budget and then show that it is being replicated daily for the week, **or** can do a full seven-day budget with water usage varying each day

▶ Provide evidence to justify budget

▶ Communicate solution in context

There are many possible solutions; a couple of solutions are given.

Solution 1

Daily Water Usage

Activity/Appliance	Daily Usage	Total	Explanation
Shower	5 min/day 5 × 8 L/min = 40 L	40 L	One quick shower per day is all that is needed
Toilet	3 flushes/day 3 × 6 L = 18 L	18 L	Assuming you work or go to school during the day you only average using the toilet 3×/day
Tap	10 min/day 10 × 6 L/min = 60 L	60 L	After using washroom, and once for dishes that can't go in dishwasher
Dishwasher	Once every second day	8L	One load of dishes per two days
Clothes Washer	Two loads/week (/7)	17 L	Only use a light and dark load per week
TOTAL		143 L/day	

143 L/day × 7 days = 1001 L for the week, which is under the limit of 1050 L per week.

Solution 2

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Shower/bath	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L	5 min shower 5 min × 8 L/min = 40 L
Toilet	3 × 6 L = 18 L	3 × 6 L = 18 L	3 × 6 L = 18 L	3 × 6 L = 18 L	3 × 6 L = 18 L	5 × 6 L = 30 L	5 × 6 L = 30 L
Tap	8 min × 6 L/min = 48 L	10 min × 6 L/min = 60 L	8 min × 6 L/min = 48 L	10 min × 6 L/min = 60 L	8 min × 6 L/min = 48 L	10 min × 6 L/min = 60 L	8 min × 6 L/min = 48 L
Dishwasher	1 × 16 L = 16 L		1 × 16 L = 16 L		1 × 16 L = 16 L		1 × 16 L = 16 L
Clothes washer		1 cycle = 60 L				1 cycle = 60 L	
Total	**122L**	**178L**	**122L**	**118L**	**122L**	**190L**	**134L**

Weekly total: 122 + 178 + 122 + 118 + 122+ 190 + 134 = 986 L
Weekly total is 986 L which is well under the total of 1050 L per week needed. This leaves room for any unexpected water usage.

Showers were taken daily. Toilet usage was increased on the weekend as I would not be using the facility at school as I do during the week.

The dishwasher is used every second day, so on the days it doesn't run there are an extra 2 minutes of tap usage to accommodate having to wash a couple of dishes by hand.

Two loads of clothes washing per week are enough to have one load of lights and one load of darks.

This keeps the water usage under the limit of 1050 L per week.

Exemplar #1 – Score: 4

☑ The work demonstrates a proficient understanding of the situation.
- water usage is identified and explained
- all work is shown

☑ The strategy is effective and comprehensive.
- calculations are clear and correct for each appliance/fixture

☑ The logic references all aspects of the problem.
- each fixture use has a reasonable explanation
- each fixture is mentioned

☑ The reasoning is clear, detailed, and organized.
- the total water usage is within the allowable amount
- communication is clear

Show your work and write your final answer in the space provided.
Montrez votre travail et écrivez votre réponse finale dans l'espace prévu.

Question 1 Topic / Questione 1 sujet

Water Use

Showers take around 8 minutes so 8L × 8min = 64L every shower. Taking a shower every 2 days in a week would be 3-4 days of showering. 64L × 3days = 192L every week.

Dishwasher is used a few times every week so 16L × 3 = 48L every week

Around 4 toilet breaks every day so 24L used everyday. 24L × 7days = 168L each week.

The tap is used around 10minutes each day so 6L × 10min = 60 then multiplied by 7 to get 420L every week.

The Clothes Washer is turned 2-3 times each week so 60L × 3 = 180L each week.

The total amount of water used is 1008L/week.

More workspace available. / Espace disponible également au verso. ➡

Exemplar #2 – Score: 4

☑ The work demonstrates a proficient understanding of the situation.
- water usage is identified and explained
- all work is shown

☑ The strategy is effective and comprehensive.
- calculations are per fixture per week
- calculations are clear and correct for each appliance/fixture

☑ The logic references all aspects of the problem.
- each fixture is mentioned

☑ The reasoning is clear, detailed, and organized.
- the total water usage is within the allowable amount
- communication is clear

Show your work and write your final answer in the space provided.
Montrez votre travail et écrivez votre réponse finale dans l'espace prévu.

water usage

Know!
≤ 1050L
Have to use
everything
1 time

I will use
974L of
water with
my one
week plan

Toilet 5 times A day for the week
$6L \times 5 \times 7 = 210 L/week$

Shower 7 times for longs for the week
$8 \times 10 \times 3 = 240L$

Tap 6L per min So 3 mins every day
$6 \times 3 \times 7 = 126$

bath 1 per week
$50L \times 1 = 50L$

Dishwasher 3 times per week
$16L \times 3 = 48L$

Cloths wash 5 times a week
$60L \times 5 = 300L$

```
  210+
  240
  450
  126+
  576+
   50
  626+
   48+
  674
  300
  474
```

More workspace available. / Espace disponible également au verso. ➡

Exemplar #3 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
  • water usage is identified
  • work is shown

☑ The strategy is sensible but has some inconsistencies.
  • most fixtures are calculated per day x 7 for the week
  • use of dishwasher and bath weekly total is not explained

☑ The logic references all aspects of the problem.
  • each fixture is used

☑ The reasoning is evident.
  • most supporting calculations are shown
  • water usage is within the allowable amount

$$1W = 7 \times 150 = 1050 L$$

Per week

5 Flushes a day = 30L ×7 = 210L
10 min. tap. ÷ a day = 60 × 7 = 420L
5 showers 5min = 40×5 = 200L
1 load of laundry = 60L
Diswasher per day = 112L
Bath = 50L

= 1052 L

Exemplar #4 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
- end result meets the criteria

☑ The strategy is sensible but has some inconsistencies.
- appropriate strategy is used but the daily consumption does not match the weekly consumption

☑ The logic references most aspects of the problem.
- incomplete justification (not all calculations are shown)
- running the dishwasher and clothes washer once a day is not logical

☑ The reasoning is evident.
- most supporting calculations are shown
- communication is clear and organized

B/w Day	Why frequencies chosen	
4 minutes a day - 32 L a day	one shower a day, for only 4 minutes	shower / bath
2 flushes a day, 22L	avg. person only needs to flush the toilet twice	toilet
5 minutes a day, 30L a day	The bathroom tap can be conserved to 2 minutes leaving 3 for the kitchen	taps (bathroom and kitchen)
1 cycle a day, 16L a day	The dishwasher only needs to be run at the end of the day	dishwasher
1 cycle a day, 60L	The laundry needs to be done at the end of the day ensuring only one cycle	clothes washer

By reducing shower time and faucet time one can use 150L of water a day, baths are not needed

overall consumption in a week = 1050

overall consumption in a day = 150

Exemplar #5 – Score: 2

☑ The work demonstrates a basic understanding of the situation.
   - incomplete mathematical solution

☑ The strategy is unclear and/or incomplete.
   - appropriate strategy selected, but not followed through

☑ The logic correctly references some aspects of the problem.
   - all fixtures are included, but total use is not calculated

☑ The reasoning is clear but incomplete.
   - supporting calculations are not shown
   - the work shown has some structure but no real solution is presented

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Shower/Bath	1 Bath / 80 L	∅	1 Bath	∅	1 Bath	∅	1 Bath
Toilet	18 L / 3 flush	3 flush	3 flush	3 flush	3 flush	3 flush	3 flush
Taps	13.6 min / 81.6 L	9.3 min	13.6 min	9.3 min	13.6 min	9.3 min	13.6
Dish-washer	∅	16 L	∅	16 L	∅	16 L	∅
Clothes Washer	∅	60 L	∅	60 L	∅	60 L	∅

Exemplar #6 – Score: 2

☑ The work demonstrates a basic understanding of the situation.
- tries to answer the question without any explicit mathematical calculations

☑ The strategy is unclear and/or incomplete.
- while usage is given for each utility, there are no calculations
- the water usage is inferred in the work, but the actual usage is never calculated or given

☑ The logic correctly references some aspects of the problem.
- each utility is addressed, but without water usage given

☑ The reasoning is unclear.
- the solution has some structure even though supporting calculations are not represented
- while there is some attempt to logically address the problem, the end usage is not given
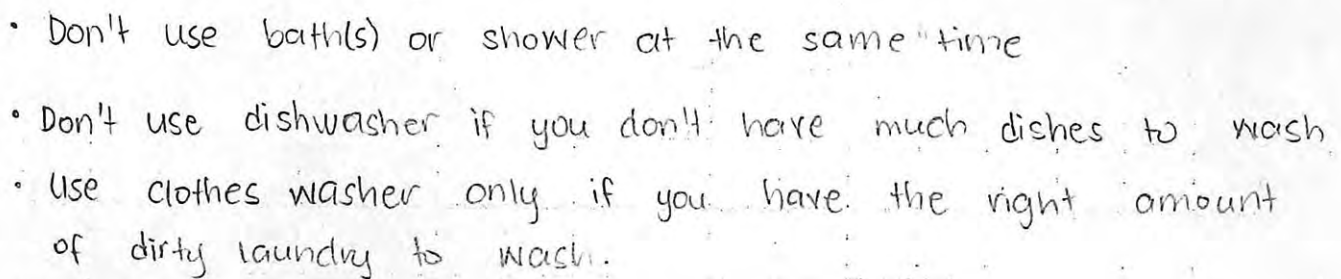
Show your work and write your final answer in the space provided.
Montrez votre travail et écrivez votre réponse finale dans l'espace prévu.

		Water Use
13. Shower	7 times a week	This comes well under
Toilet	25 times a week	the budget even we aren't
Tap	40 times a week	
Dishwasher	7 times a week	using these resources too
Clothes Washer	5 times a week	frugantly. You can take
		a shower a day and
		run the dishwasher
		when full.

Exemplar #7 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
  - addresses the concept of reduced water usage

☑ The strategy is unclear and/or incomplete.
  - while there is a limited attempt at reducing water usage, there are no calculations

☑ The logic is inadequate.
  - no mathematical calculations at all
  - only three utilities mentioned

☑ The reasoning is incomplete.
  - no calculations are shown
  - there is no solution

- Don't use bath(s) or shower at the same time
- Don't use dishwasher if you don't have much dishes to wash
- Use clothes washer only if you have the right amount of dirty laundry to wash.

Exemplar #8 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
  • there is no mention of reducing water usage

☑ The strategy is ineffective.
  • strategy is unclear
  • there are no calculations

☑ The logic does not reference the problem.
  • the bar graph is not labelled and there is no mention of the various utilities

☑ The reasoning is missing.
  • supporting calculations are not shown
  • there is no solution given
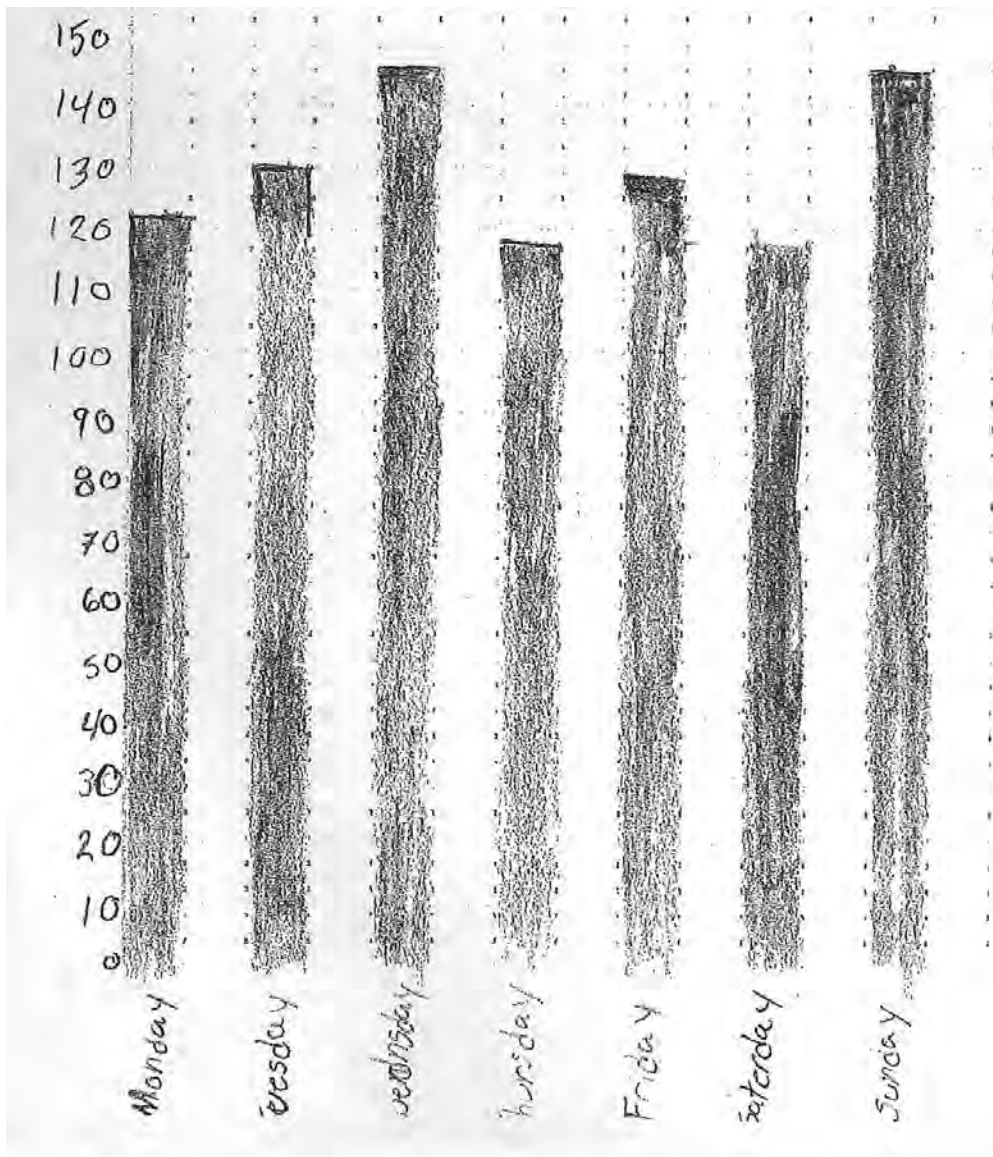
Exemplar #9 – Score: 0

☑ The response does not address the purpose of the task.
  • there is no explanation of the calculation, and no solution given

$$\frac{11.15 \text{ min}}{} \times 5 \times 7 = \frac{390.25 \text{ min}}{\text{week}}$$

$$\times 10 = \frac{3,902.5 \text{ L}}{\text{week}}$$

Btw my hand is broken so that is why my writing is bad.

Understanding Our Past: Pit Houses

Archaeologists study artifacts and monuments to understand past cultures. In the interior of British Columbia, First Peoples lived in circular homes, called pit houses. Pit houses varied in size, depending on how many people lived in the home.

1 m^2
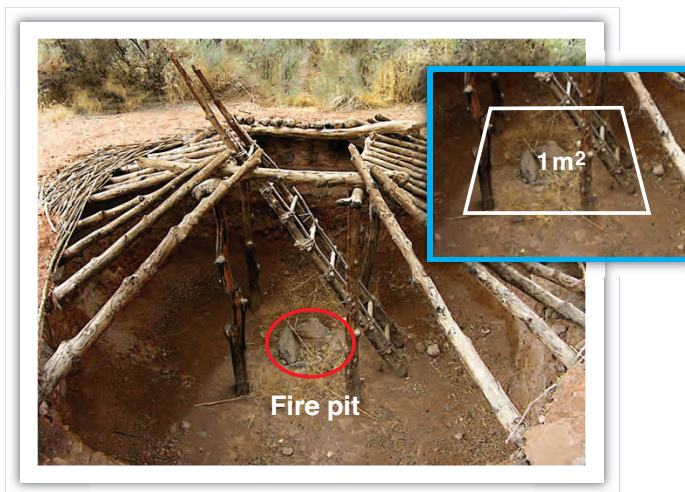
Fire pit

Approximate Floor Area Required

Family

Area: 10 m^2

Couple

Area: 7 m^2

Adult

Area: 4 m^2

Fire Pit and Ladder
Area: 1 m^2

Overhead Views

Covered
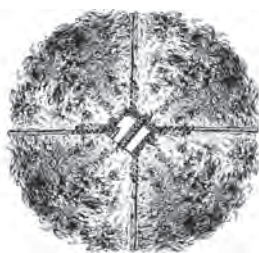
Structure
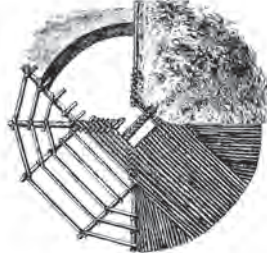
13. Archaeologists examine the remains of a village of pit houses. Impressions of where the pit houses were located are still visible. The diagram below shows the remains of a village with a number of pit houses.
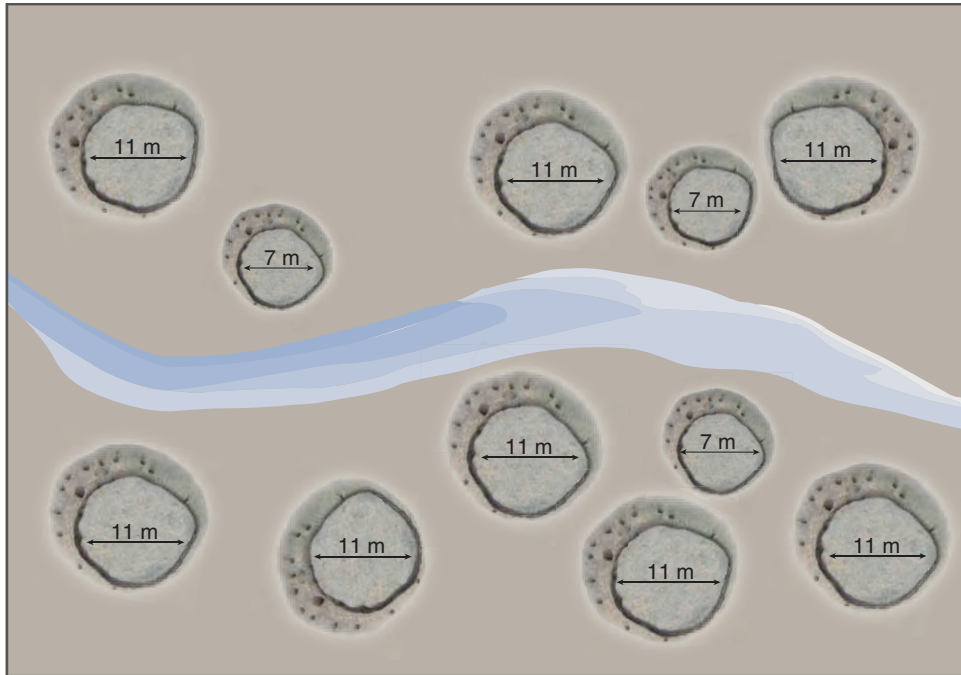
Estimate the number of people that could have lived in this village. State any assumptions made.

Explain and justify your solution.

This question is to be answered on paper.

Summary of Requirements for a Level 4:

▶ State initial assumptions
▶ Calculate area of small and large pit houses
▶ Estimate number of people
▶ Provide evidence to justify estimate
▶ Communicate solution in context

There are many possible solutions; several solutions are given.

Solution 1

Assumptions:

- 1 family consisting of 2 adults and 2 children requires 10 m^2 of living space
- ladder and fire pit require an area of 1 m^2 per pit house
- the same configuration of people live in each of the small and large pit houses
- all pit houses are occupied to their full capacity

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

Area of a small pit house:

$A = \pi r^2, \ r = \dfrac{d}{2}$

$A = \pi (3.5)^2$

$A \approx 38.48 \ m^2$

Area available for living:

38.48 m^2 – 1 m^2 (for fire pit and ladder) $\approx$ 37 m^2

Area of a large pit house:

$A = \pi r^2, \ r = \dfrac{d}{2}$

$A = \pi (5.5)^2$

$A \approx 95.03 \ m^2$

Area available for living:

95.03 m^2 – 1 m^2 (for fire pit and ladder) $\approx$ 94 m^2

Number of people who lived in a small pit house:

$\dfrac{37 \ m^2}{10 \ m^2/family} \approx 3$ families

Area required for 3 families: 3(10 m^2) = 30 m^2

37 m^2 – 30 m^2 = 7 m^2 left over

Area required for 1 couple = 7 m^2

Total number of people who lived in a small pit house: 3(4) + 1(2) = 14

Since there are 3 small pit houses: 3(14) = 42

Number of people who lived in a large pit house:

$\dfrac{94 \ m^2}{10 \ m^2/family} \approx 9$ families

Area required for 9 families: 9(10 m^2) = 90 m^2

94 m^2 – 90 m^2 = 4 m^2 left over

Area required for 1 adult = 4 m^2

Total number of people who lived in a large pit house: 9(4) + 1 = 37

Since there are 8 large pit houses: 8(37) = 296

Total number of people: 42 + 296 = 338

The estimated number of people who lived in the village is 338. This is reasonable under the above-mentioned assumptions. This solution also assumes that people are evenly distributed amongst the pit houses and that the maximum living space within each pit house is fully utilized.

Solution 2

Assumptions:

- 1 adult needs 4 m² of living space
- 1 couple needs 7 m² of living space
- 1 family consisting of 2 adults and 2 children requires 10 m² of living space
- ladder and fire pit require an area 1 m² per pit house
- the same configuration of people live in each of the small and large pit houses
- the maximum amount of living space in each pit house is used

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

Area of a small pit house:

$A = \pi r^2, \ r = \dfrac{d}{2}$

$A = \pi (3.5)^2$

$A \approxeq 38.48 \text{ m}^2$

Area available for living:

38.48 m² − 1 m² (for fire pit and ladder) ≈ 37 m²

Area of a large pit house:

$A = \pi r^2, \ r = \dfrac{d}{2}$

$A = \pi (5.5)^2$

$A \approxeq 95.03 \text{ m}^2$

Area available for living:

95.03 m² − 1 m² (for fire pit and ladder) ≈ 94 m²

Strategy: Guess and check:

Number of people living in small pit houses:

Area required for 2 families: (2)(10 m²) = 20 m²

Area required for 4 adults: (4)(4 m²) = 16 m²

Area required is 20 m² + 16 m² = 36 m², which is less than 37 m² that is available.

Number of people: (2)(4) + 4 = 12 people per house

∴ (3)(12) = 36 people lived in the 3 small pit houses.

Number of people living in large pit houses:

Area required for 6 families: (6)(10 m²) = 60 m²

Area required for 3 adults: (3)(4 m²) = 12 m²

Area required for 3 couples: (3)(7 m²) = 21 m²

Area required is 60 m² + 12 m² + 21 m² = 93 m², which is less than 94 m² that is available.

Number of people: (6)(4) + 3 + (3)(2) = 33 people per house

∴ (8)(33) = 264 people lived in the 8 large pit houses.

36 + 264 = 300 people

The estimated number of people living in the village is 300. This is a reasonable estimate under the above-mentioned assumptions. However, some families may have more children than 2, thus requiring more space. On the other hand, some families may have had only one child thus requiring less space.

Solution 3

Assumptions:

- 1 adult needs 4 m² of living space
- 1 couple needs 7 m² of living space
- 1 family consisting of two adults and two children requires 10 m² of living space
- ladder and fire pit require an area of 1 m² per pit house
- different configurations of people live in each of the small pit houses and a few of the large pit houses

There are 11 pit houses – 3 small with diameter of 7 m and 8 large with diameter of 11 m.

Area of a small pit house:

$A = \pi r^2, \ r = \dfrac{d}{2}$

$A = \pi (3.5)^2$

$A \approx 38.48$ m²

Area available for living:

38.48 m² – 1 m² (for fire pit and ladder) $\approx$ 37 m²

Area of a large pit house:

$A = \pi r^2, \ r = \dfrac{d}{2}$

$A = \pi (5.5)^2$

$A \approx 95.03$ m²

Area available for living:

95.03 m² – 1 m² (for fire pit and ladder) $\approx$ 94 m²

Number of people living in the 3 small pit houses:

Pit House 1:

    Area required for 2 families + 4 adults: (2)(10 m²) + (4)(4 m²) = 36 m²

    (2)(4) + 4 = 12 people

Pit House 2:

    Area required for 5 couples: (5)(7 m²) = 35 m²

    (5)(2) = 10 people

Pit House 3:

    Area required for 1 family + 2 couples + 3 adults: (1)(10 m²) + (2)(7 m²) + (3)(4 m²) = 36 m²

    (1)(4) + (2)(2) + 3 = 11 people

Total number of people who lived in the small pit houses: 12 + 10 + 11 = 33 people.

Strategy: Guess and Check
Number of people living in the 11 large pit houses:

5 large pit houses occupied by families only – 9 families in each house

    Area required for 9 families: (9)(10 m²) = 90 m²

    (9)(4) = 36 people in one large pit house   (36)(5) = 180 people in 5 large pit houses

3 large pit houses occupied by 11 adults and 7 couples:

    Area required for 11 adults and 7 couples: (11)(4 m²) + (7)(7 m²) = 44 m² + 49 m² = 93 m²

    11 + (7)(2) = 25 people   (25)(3) = 75 people

Total number of people who lived in the large pit houses: 180 + 75 = 255 people.   33 + 255 = 288 people

The estimated number of people living in the village is 288. This is a reasonable estimate under the above-mentioned assumptions. However, some families may have more children than 2, thus requiring more space. On the other hand, some families may have had only one child, thus requiring less space.

Solution 4

Assumptions

- 1 adult needs 4 m^2 of living space
- 1 couple needs 7 m^2 of living space
- 1 family consisting of 2 adults and 2 children requires 10 m^2 of living space
- ladder and fire pit require an area of 1 m^2 per pit house
- considering the total area of living space across all pit houses rather than by individual pit house

There are 11 pit houses – 3 small with diameter of 7 m and 8 large ones with diameter of 11 m.

Area of a small pit house:

$A = \pi r^2, \quad r = \dfrac{d}{2}$

$A = \pi (3.5)^2$

$A \approx 38.48 \ \text{m}^2$

Area available for living:

38.48 m^2 – 1 m^2 (for fire pit and ladder) $\approx$ 37 m^2

Area of a large pit house:

$A = \pi r^2, \quad r = \dfrac{d}{2}$

$A = \pi (5.5)^2$

$A \approx 95.03 \ \text{m}^2$

Area available for living:

95.03 m^2 – 1 m^2 (for fire pit and ladder) $\approx$ 94 m^2

Total area available for living: (3)(37 m^2) + (8)(94 m^2) = 863 m^2

Number of people living in the pit houses:

$\dfrac{863 \ \text{m}^2}{10 \ \text{m}^2/\text{family}} \approx 86 \text{ families}$

$\therefore$ 86 families can live in this area

Area required for 86 families: 86(10 m^2) = 860 m^2

863 m^2 – 860 m^2 = 3 m^2 left over

3 m^2 which is not enough room for a single adult or couple to live

Total number of people: 86(4) = 344 people

Therefore, my estimate would be 344 people lived in the village.

The estimate of 344 people is reasonable under the above-mentioned assumptions. However it is also likely that there were couples and single adults living in this village. Considering the total area of living space across all pit houses rather than by individual pit house made my calculations faster with fewer steps. However, this assumption may not properly reflect the real-life scenario if the houses have a considerable amount of space unoccupied.

Exemplar #1 – Score: 4

✓ The work demonstrates a proficient understanding of the situation.
- combined the area requirements to efficiently fill available space in each pit house
- assumptions are evident: the area combinations reflect the context of a village – population consists of families, couples, and adults

✓ The strategy is effective and comprehensive.
- all area requirements are reflected in the solution
- the total population is based on the distributions of people in each size of pit house

✓ The logic references all aspects of the problem.
- determined the available floor area for each size of pit house [area = π(radius)^2]
- used the area requirements to determine the number of people that could occupy each size of pit house
- provided a total population for the village based on the number of each size of pit house

✓ The reasoning is clear, detailed, and organized.
- areas allocations are shown for each size of pit house
- distributions of people are shown for each size of pit house
- total area and total population calculations are evident

Assumptions:
- families need 10m² living space
- Couples need 7m² living space
- Adults need 4m² living space
- there will be a combination of families, couples and adults in the community
- fire/ladder requires 1m²

Small pit house
$A = \pi r^2$ $A = \pi(3.5)^2$ $A = 38.48 m^2$
38m² – 1m² for fire/ladder = ~37 m² of living space

- Families will be the most efficient use of space,
so $\frac{37 m^2}{10m^2 \text{ for family}}$ = 3 families plus 7m² left of space

7m² is enough room for a couple
So a small pit house can fit 3 families and 1 couple which is 14 people

There are 3 small pit houses. 3×14 = 42 people

Large pit house area
$A = \pi r^2$ $A = \pi(3.5)^2$ $A = 95.03m^2$
95m² - 1m² for firepit = 94 m²
If we assume that about half of the pit houses are for families and the other half are for adults and couples, then we can assume about 5 families live in the pit house, taking up 50m² living space.
94m² - 50 m² = 44 m² left over
5 couples × 7m² space = 35m²
44m² - 35m² = 9m² left which is enough space for 2 adults with 1m² left
Therefore 1 large pit house can hold
5 families + 5 couples + 2 adults
5(4) + 5(2) + 2 =
20 + 10 + 2 = 32 people
8 large pit houses × 32 people = 256 people
small pit houses + large pit houses
42 + 256 = 298
I estimate about 298 people lived in the village, using the stated assumptions.

Exemplar #2 – Score: 4

☑ **The work demonstrates a proficient understanding of the situation.**
- combined various area requirements to efficiently fill available space in each pit house
- assumptions are evident: the area combinations reflect the context of a village – population consists of families, couples, and adults

☑ **The strategy is effective and comprehensive.**
- all area requirements are reflected in the solution
- the total population is based on the distributions of people in each size of pit house

☑ **The logic references all aspects of the problem.**
- determined the available floor area for each size of pit house [area = π(radius)^2]
- used the area requirements to determine the number of people that could occupy each size of pit house
- provided a total population for the village based on the number of each size of pit house

☑ **The reasoning is clear, detailed, and organized.**
- areas allocations are shown for each size of pit house
- distributions of people are shown for each size of pit house
- total area and total population calculations are evident

assumptions
- Families are the most likely to live in the community so most pit houses will have families.
- Each family requires 10m² of living space
- Each pit house has a 1m² firepit

Small pit house)
$A = \pi\left(\frac{7}{2}\right)^2 = 38.48 m^2$
38.48 × 3 houses = 115.45m² - 3(1m²) fire = 112.45m²
$\frac{112.45 m^2}{10 m^2}$ = ≈ 11 families with 2.45m² leftover
11 × 4 people = 44 people

Large pit houses
$A = \pi\left(\frac{4}{2}\right)^2$ A = 95.03 m²
95.03 × 8 houses = 760.24m²
760 m² - 8m² firepit = 752m²
752 ÷ 10m² = ~75 families
75 × 4 people = 300 people in large houses

300 + 44 = 344 people

I estimate there will be 344 people in the village. This is an estimate because I combined all the living space together, and the actual pit houses might not fit this many, also, there would probably be single adults and couples in the village, and they take more space, so the real numbers is probably less than 344.

Exemplar #3 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
- used one of the area requirements to fill available space in each pit house
- assumptions were implied, but not clearly stated: the area combinations reflect adults only (no couples or children in the village)

☑ The strategy is sensible but has some inconsistencies.
- only one area requirement is reflected in the solution
- the total population is based on the distributions of adults only in each size of pit house

☑ The logic references all aspects of the problem.
- determined the available floor area for each size of pit house [area = π(radius)²]
- used the area requirements to determine the number of adults that could occupy each size of pit house; fire pit requirements were not included; couples and children were not included
- determined a total number of adults for the village based on the number of each size of pit house

☑ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- only one area allocation is shown for each size of pit house
- distributions of people are shown for each size of pit house (but did not change allocations to make efficient use of different areas)
- total area and total population calculations are evident

Adults require 4m² of living space, 11m diameter houses hold 23 adults, 7m diameter houses hold 9 adults, for a total of 211 Adults that lived in the village.

$\pi r^2 = A$	11m diameter bases
$\pi(\frac{11}{2})^2 = A$	95m²
$95.0 = A$	$\frac{95m^2}{4m^2} = 23.758$ adults = 23 adults

$\pi r^2 = A$	7m diameter houses
$\pi(\frac{7}{2})^2 = A$	38.5m²
$38.5 = A$	$\frac{38.5m^2}{4m^2} = 9.62$ adults = 9 adults

8 – 11m	23×8 = 184 adults
3 – 7m	9×3 = 27 adults
	211 adults in the village

Exemplar #4 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
- used one of the area requirements to fill available space in each pit house
- assumptions were implied, but not clearly stated: the area combinations reflect each group, but not a combination of people

☑ The strategy is sensible but has some inconsistencies.
- only one area requirement is reflected in the solution
- the total population is based on the distributions of adults OR families OR couples – not a combination

☑ The logic references all aspects of the problem.
- determined the available floor area for each size of pit house [area = $\pi(radius)^2$]
- used the area requirements to determine the number of people that could occupy the total area of all houses combined; fire pit requirements were not included
- determined a total number of people for the village based on the total area of all houses

☑ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- only one area allocation is shown for each size of pit house
- distributions of people are shown for the total area of all houses (but did not consider allocations to make efficient use of different areas)
- total area and total population calculations are evident

$$A = \pi r^2$$

$$A = \pi 5.5^2 \times 8$$

$$A = 94.985 \times 8$$

$$A = 759.88 m^2$$

$$A = \pi 3.5^2 \times 3$$

$$A = 115.395 m^2$$

$$A_{tot} = 875.275$$

215 adults could live in the village plus 1.1 ladder/firepits
86 families could live in the village plus 1.1 ladders
123 couples could live plus 11 ladders

Exemplar #5 – Score: 2

☑ The work demonstrates a basic understanding of the situation.
- used one of the area requirements area requirements to fill available space in each pit house
- incorrect assumptions: the area combinations reflect adults only (no couples or children in the village)

☑ The strategy is unclear and/or incomplete.
- only one area requirement is reflected in the solution
- the total population is based on the distributions of adults only in the total area of all houses

☑ The logic correctly references some aspects of the problem.
- incorrectly determined the available floor area for each size of pit house (used the diameter instead of the radius)
- used the area requirements to determine the number of adults that could occupy each size of pit house; fire pit requirements were not included; couples and children were not included
- determined a total number of adults for the village based on the number of each size of pit house

☑ The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
- only one allocation is shown for the total area of the all houses combined
- distributions of people were the same for each size of pit house (no consideration made for efficient use of different areas)
- total area and total population calculations are evident


```
assumptions :    ① the minimum space for
                    1 person is  4m²
                 ② the space required for ladder
                    and fire pit is neglected
                    (doesn't count).
                 ③ the diameter of pit houses are
                    either 11 m or 7m exact
                 ④ π equals to 3.14
total area =   8 × π × 11² + 3 × π × 7²
           =  3501. 1 m²
number of people living in the village = total area
                                         minimum space for 1
                                       = 3501. 1 m²
                                          4 m²
                                       = 875 . 28
                                       = 875  (people)
The  max number of people is 875.
```

Exemplar #6 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
- the concept of area was not applied to the situation
- assumptions not stated: unclear whether adults, couples, or children were considered to be in the village

☑ The strategy is ineffective.
- diameters referenced instead of areas
- the total population is based on incorrect distributions of people in each size of pit house

☑ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- the logic behind the values used towards a solution is not shown (allocations of areas were not considered for different combinations of people or for a fire pit)

☑ The reasoning is missing or irrelevant; supporting calculations are not shown.

$11 = houses$

$8 = 11 m$

$3 = 7m$

If the 11 meter house can fit approx.
4 people and the 7m house can fit approx
3 people... $4 \cdot 8$
      $+ \quad = 53$ people (esstimate)
      $3 \cdot 7$ can fit in this village.

Exemplar #7 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
- the concept of area was not applied to the houses
- assumptions not stated: unclear whether couples and families were referenced but not linked to the logic of the problem

☑ The strategy is ineffective.
- diameters are stated as areas
- the total population is based on incorrect distributions of people in each size of pit house

☑ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- the logic behind the values used towards a solution is not shown (allocations of areas for different combinations of people were unclear; area for the fire pit was not considered)

☑ The reasoning is missing or irrelevant; supporting calculations are not shown.

theres 8 11m² houses and 3 7m² houses

if a couple approximatly needed 7m 7m² thats

6 People if a family of 5 needed a

Space of 10m² thats 40 People 40+6=

46 people lived in the villge

Exemplar #8 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
  - the concept of area was referenced but incorrectly applied to the situation
  - assumptions not stated: unclear whether adults, couples, or children were considered to be in the village

☑ The strategy is ineffective.
  - diameters referenced instead of areas
  - the total population is based on incorrect area requirements

☑ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
  - the logic behind the values used towards a solution is not shown (allocations of areas were not considered for different combinations of people or for a fire pit)
  - final solution does not reference a sensible value for populations (0.5 of a person)

☑ The reasoning is missing or irrelevant; supporting calculations are not shown.

$$(11 \cdot 8) + (7 \cdot 3) \overset{\text{Pit house total area}}{=}$$

$$88 + 21 = 109 \text{m}^2$$

$$98 \div 4 = 24.5 \text{ people}$$

$$109 \div 11 = 98 \text{m}^2$$

24.5 people could have lived in the village

Exemplar #9 – Score: 0

☑ The work is not relevant to solving the problem.

☑ Response does not address the purpose of the task.
   - area is not referenced
   - a value is stated, but is not supported or linked to the context of the problem

☑ The solution is incorrect with no work shown.

☑ Information is simply recopied from the problem. A population value is stated with no reference or support.

27. In the village i estamate that there would be 139 people living in the pit house village

Five years ago, Jae Eun and Ted, started a company together. They each contributed a different amount of money to start the company.

CONTRACT

This contract governs the partnership between ___Jae Eun Park___ and ___Ted Brown___ (hereby known as The Founders), doing business as _"Jae Eun & Ted's Excellent Video Game Adventures"_ (hereby known as The Company).

The Company shall create video games.

Founder Jae Eun Park commits to contribute computer equipment (valued at $8500) and $50 000 towards the start-up costs.

Founder Ted Brown commits to contribute $22 000 towards the start-up costs. He will also pay $1250 per month for The Company's expenses.

By signing below, each Founder indicates acceptance of the terms of this contract in its entirety.

_____   _____
Jae Eun Park   Ted Brown

The Company created several popular video games. Now Jae Eun and Ted receive an offer from an individual who wants to buy their company.

26. Five years after the start of The Company, The Founders decide to sell it for $750 000. To determine their fair share of the sale price, they agree that any contributions made towards start-up costs will be worth 1.5 times their original value. Contributions made after start-up will not be adjusted.

    How much should Jae Eun and Ted each receive from the sale of their company?

    Explain and justify your solution.

    This question is to be answered on paper.

Summary of Requirements for a Level 4:

▶ Determine the current value of the initial investments both partners made (Jae Eun $87 750 and Ted $33 000)

▶ Calculate the total investment over 5 years (Ted $108 000 and Jae Eun $87 750)

▶ Select and communicate a strategy for sharing the $750 000

▶ Provide evidence to justify solution

▶ Communicate solution in context

There are several possible solutions; a few solutions are given.

Solution 1

This solution is based on each partner receiving a percentage of the total sale price based on what they invested in the company.

The initial investment is worth 1.5 times its original value. Jae Eun and Ted contributed different amounts at the start.

Jae Eun: $50 000 + $8 500 = $58 500
$58 500 × 1.5 = $87 750 (Jae Eun's initial contribution weighted at 1.5×)

Ted: $22 000 × 1.5 = $33 000 (Ted's initial contribution weighted at 1.5×)

Ted continued to contribute $1250/month for 5 years.

$1250 × 5 years × 12 months/year = $75 000

In total, Ted invested: $33 000 + $75 000 = $108 000

Altogether, Jae Eun and Ted contributed: $87 750 + $108 000 = $195 750

The percentage of $195 750 that each partner contributed:

$$\text{Jae Eun} \quad \frac{\$87\,750}{\$195\,750} \times 100 = 44.8\%$$

$$\text{Ted} \quad \frac{\$108\,000}{\$195\,750} \times 100 = 55.2\%$$

Therefore, Jae Eun should receive 45% (or 44.8%) of the sale price, while Ted should receive 55% (or 55.2%).

Jae Eun: $750 000 × 0.45 = $337 500 (or $336 000 using decimal %)
Ted: $750 000 × 0.55 = $412 500 (or $414 000 using decimal %)

Solution 2

This solution is based on splitting the profit of the sale after deducting the partners' contributions.

The same initial calculations as in Solution 1:

Jae Eun: $50 000 + $8 500 = $58 500
$58 500 × 1.5 = $87 750 (Jae Eun's initial contribution weighted at 1.5×)

Ted: $22 000 × 1.5 = $33 000 (Ted's initial contribution weighted at 1.5×)

Ted continued to contribute $1250/month for 5 years.

$1250 × 5 years × 12 months/year = $75 000

In total, Ted invested: $33 000 + $75 000 = $108 000

Total Partner contributions: $87 750 + $108 000 = $195 750

Sale price of Company less Total Partner contributions: $750 000 – $195 750 = $554 250

Profit split equally: $554 250 ÷ 2 = $277 125 for each partner

Therefore:
Jae Eun: $87 750 + $277 125 = $364 875
Ted: $108 000 + $277 125 = $385 125

Solution 3

This solution is based on splitting the profit of the sale and crediting the difference between investment contributions to the partner that paid more.

The same initial calculations as in Solution 1:

Jae Eun: $50 000 + $8 500 = $58 500
$58 500 × 1.5 = $87 750 (Jae Eun's initial contribution weighted at 1.5×)

Ted: $22 000 × 1.5 = $33 000 (Ted's initial contribution weighted at 1.5×)

Ted continued to contribute $1250/month for 5 years.

$1250 × 5 years × 12 months/year = $75 000

In total, Ted invested: $33 000 + $75 000 = $108 000

Ted invested more than Jae Eun: $108 000 – $87 750 = $20 250.
Ted paid $20 250 more than Jae Eun.

Split the sale income: $750 000 ÷ 2 = $375 000

As Ted invested more in the company, he will receive $20 250 more than half the sale price,
while Jae Eun will receive $20 250 less than half the sale price:

Jae Eun: $375 000 – $20 250 = $354 750
Ted: $375 000 + $20 250 = $395 250

Exemplar #1 – Score: 4

☑ The work demonstrates a proficient understanding of the situation.
- determines the total investment over 5 years for each partner
- allocates a percentage of the sale price to each partner based on their individual investment

☑ The strategy is effective and comprehensive.
- percentage of sale price allocated is based on each partner's total investment over 5 years

☑ The logic references all aspects of the problem.
- each partner's investment is considered when determining the allocation of sale price

☑ The reasoning is clear, detailed, and organized.
- the calculations for investment and percentage contribution are included and correct
- the allocation of sale price is determined and a rationale is provided

Jae: $58500 × 1.5 = $87,750

Ted: $22000 × 1.5 = $33000 + $1250 × 12 months × 5 years = 75000

Total = $108,000

per percentage contribution: Total = $195,750

% Jae = $\frac{$87,750}{$195,750} × 100 = 44.8\%$

% Ted = $\frac{$108,000}{$195,750} × 100 = 55.2\%$

compensation:

Jae: $750,000 × 0.448 = $336,000

Ted: $750,000 × 0.552 = $414,000

Jae should recieve 44.8% of the money from the sale. Berause that is the percent he contributed to the company in funds. This means that Jae should recieve $336,000. Ted should recieve 55.2% of the money from the sale. The reason for this is because he contributed 55.2% to the company when it comes to money, so in theory he owns 55.2% of the company so he should recieve $414,000 of the $750000

Exemplar #2 – Score: 4

☑ The work demonstrates a proficient understanding of the situation.
- determines the total investment over 5 years for each partner
- determines that each partner should be allocated back their individual investments and then half of what remains from the sale price

☑ The strategy is effective and comprehensive.
- from the sale price, each partner receives back their total investment over 5 years and then half of what remains

☑ The logic references all aspects of the problem.
- each partner's investment is considered when determining the allocation of sale price

☑ The reasoning is clear, detailed, and organized.
- the calculations for investment are included and correct
- the allocation of sale price is determined and is reasonable

$750,000

Ted	Jae
1250 per month	8500 computers x1.5
22 000 kickstart x1.5	50 000 startup x1.5
37,000	12 750
+ 75 000	75 000
108 000 total contributions	87 750 total contributions

```
  750 000
- 108 000
  642 000
-  87 750
  554 250
÷       2
  277 125
```

Ted
```
  108 000
+ 277 125
$ 385 125
```

Jae
```
   87 750
+ 277 125
$ 364 875
```

Ted should recieve $385 125.

Jae should recieve $ 364 875.

I think they should each be payed back what they contributed and then have the rest of the money split in half.

Exemplar #3 – Score: 4

☑ The work demonstrates a proficient understanding of the situation.
- determines the total investment over 5 years for each partner
- allocates a percentage of the sale price to each partner based on their individual investment

☑ The strategy is effective and comprehensive.
- percentage of sale price allocated is based on each partner's total investment over 5 years

☑ The logic references all aspects of the problem.
- each partner's investment is considered when determining the allocation of sale price

☑ The reasoning is clear, detailed, and organized.
- the calculations for investment and percentage contribution are included and correct
- the allocation of sale price is determined and supported by sound mathematical reasoning

Jae contribution = 58,500 × 1.5
= 87,750 $

Ted Contribution = 22,000 × 1.5 + 60 × 1250   195,750
33,000 + 75,000
108,000 $

108,000 : 87,750
Ted contribute 55%
Jae contribute 45%

Ted gets 412,500 $
Jae gets 337,500 $
because of percentage
of previous
contribution

Exemplar #4 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
- determines the total investment over 5 years for each partner
- allocates a percentage of the sale price to each partner based on their individual investment

☑ The strategy is sensible but has some inconsistencies.
- percentage of sale price allocated is based on each partner's total investment over 5 years

☑ The logic references all aspects of the problem.
- each partner's investment is considered when determining the allocation of sale price

☑ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- the calculations for investment are included, but contain an error
- the allocation of sale price is determined

$$50000 + 8500 = 58,500 \times 1.5 = 87,750 = Jae$$
$$22000 + 1250 \times 12 \times 5 = 97000 \times 1.5 = 145,500 = Ted$$
$$750\,000 - (87,750 + 145,500) =$$
$$750,600 - 233,250 = 516,750$$
$$516,750 \div 2 = 258,375$$

Ted and Jae should both get $258,375 from selling the company as well as 87,750 for Jaes contributions and 145,500 for Teds contributions

Exemplar #5 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
- determines the total investment over 5 years for each partner; however, start-up cost adjustment is not applied to one partner's contribution
- allocates a percentage of the sale price to each partner based on their individual investment

☑ The strategy is sensible but has some inconsistencies.
- percentage of sale price allocated is based on each partner's total investment over 5 years

☑ The logic references all aspects of the problem.
- each partner's investment is considered when determining the allocation of sale price

☑ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- the calculations for investment are included, but contain an error
- the allocation of sale price is determined and a rationale is provided

Sold for 750,000 after 5 years

Jae Eun - 58,500 × 1.5 = 87,750 (50,000 + 8,500)

Ted - 97,000 (22,000 + (1250 × 12 × 5))

$58,500 + 97,000 = \boxed{184,750}$ = money put in (total)

$Jae\ Eun = \frac{87,750}{184,750} = \boxed{0.47}$

$Ted = \frac{97,000}{184,750} = \boxed{0.52}$ → Their shares of the company

Jae = 750,000 × 0.47 = 352,500

Ted = 750,000 × 0.52 = 390,000

Jae Eun should receive $352,500 of the profit because he contributed 0.47 of the money to the company. Ted should receive $390,000 of the profit because he contributed 0.52 of the money to the company.

Exemplar #6 – Score: 2

✓ The work demonstrates a basic understanding of the situation.
- determines the total investment over 5 years for each partner; however, calculations contain an error
- a portion of the sale price is allocated to each partner; however, the rationale for the allocation is unclear

✓ The strategy is unclear and/or incomplete.
- although a portion of the sale price is allocated to each partner, it is unclear how that amount was determined

✓ The logic correctly references some aspects of the problem.
- total investment of each partner is determined and each partner is allocated a portion of the sale price
- each partner's investment has some bearing on the sale price allocation; however, a significant proportion of the sale price is not allocated

✓ The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
- the calculations for investments are shown, but contain an error
- the calculations to determine sale price allocation are shown; however, rationale is not provided and amounts are unreasonable

Exemplar #7 – Score: 2

☑ The work demonstrates a basic understanding of the situation.
- states the total investment over 5 years for each partner; however, supporting calculations are missing
- a percentage of the sale price is allocated to each partner; however, there is no rationale to the allocation

☑ The strategy is unclear and/or incomplete.
- although a percentage of the sale price is allocated to each partner, it is unclear how that percentage was determined

☑ The logic correctly references some aspects of the problem.
- total investment of each partner is stated and each partner is allocated a percentage of the sale price
- each partner's investment has no bearing on the sale price allocation

☑ The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
- the calculations for investment are not shown
- no rationale is provided for how percentage allocation was determined

Exemplar #8 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
- each partner's investment is not addressed
- half of the sale price is allocated to each partner; however, there is no rationale to the allocation

☑ The strategy is ineffective.
- although half of the sale price is allocated to each partner, it is not evident how that portion was determined

☑ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- each partner's investment has no bearing on the sale price allocation

☑ The reasoning is missing or irrelevant; supporting calculations are not shown.

750 000 ÷ 2 = 375,000

They should get 375,000 dollars each.

Exemplar #9 – Score: 1

✓ The work demonstrates an inadequate understanding of the situation.
- start-up cost adjustment is applied to each partner's initial investment; however, total investment over 5 years is not calculated
- a portion of the sale price is allocated to each partner; however, there is no rationale to the allocation

✓ The strategy is ineffective.
- although a portion of the sale price is allocated to each partner, it is incorrect given the context of the problem
- a significant proportion of the sale price is not allocated, with no rationale provided

✓ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- although each partner's initial investment has some bearing on the sale price allocation, the allocation is illogical

✓ The reasoning is missing or irrelevant; supporting calculations are not shown.

$$22,000 \times 1.5 = 33,000$$
$$58,500 \times 1.5 = 87,750$$
$$+ \underline{}$$
$$120,750$$

They will each receive $120,750

$$750,000 \div 120,750 =$$

Exemplar #10 – Score: 0

☑ An incorrect mathematical solution with no work shown.

Ted : $450,000

Jae Eun: $300,000
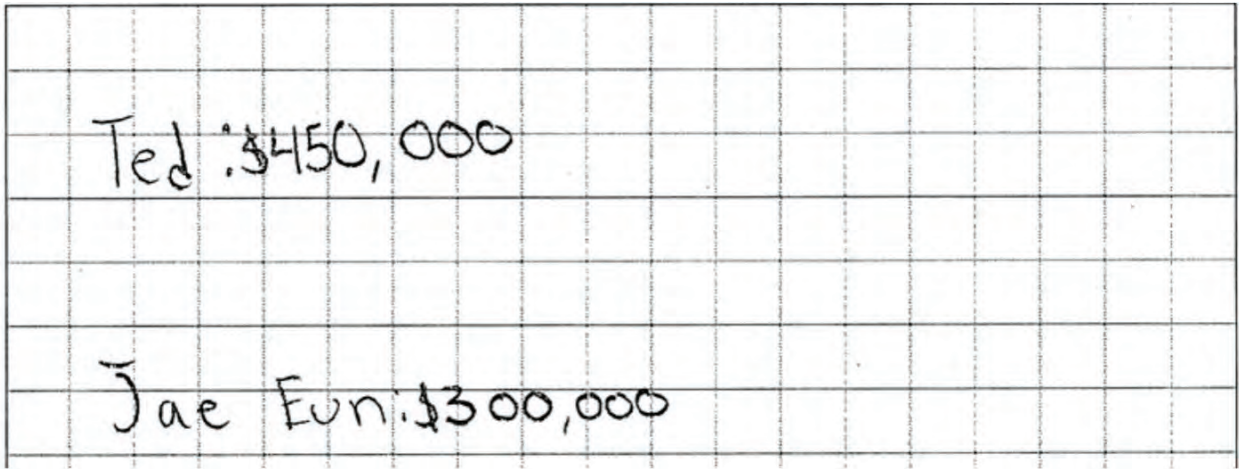
You are being trained in managing forest fires. Your Fire Fighting Training Manual describes a forest fire spread simulation.

NORTH REGION
Fire Fighting Training Manual
Forest Fires: How They Spread

The map below is a fire grid. It describes the likelihood of a fire spreading to different cells.

Fire may spread from one cell into neighbouring cells as shown:

2	1	0	1	1
2	1	0	1	0
1	1	0	0	0
1	0	2	2	2
1	1	2	1	2
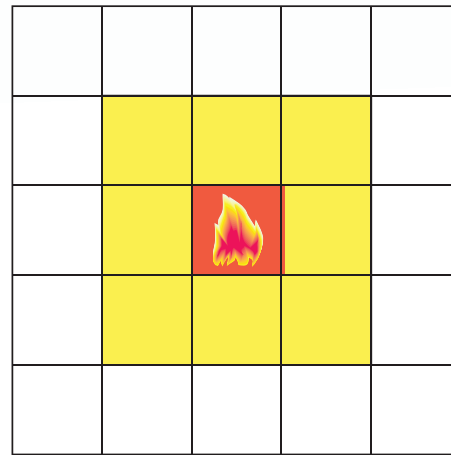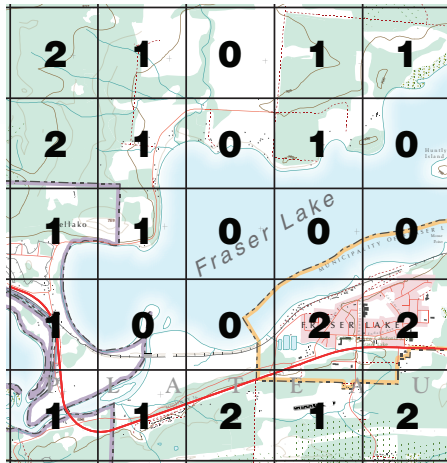
■ Burning cell   ■ Neighbouring cell

Each cell has the following fire-spread rating reflecting the probability that fire will spread to that cell from a neighbouring cell:

Fire-Spread Rating	Probability of Spreading
0	Fire has a 0% chance of spreading into it from neighbouring cells
1	Fire has a 50% chance, or probability of 0.5, of spreading into it from neighbouring cells
2	Fire has an 80% chance, or probability of 0.8, of spreading into it from neighbouring cells

Example: At time zero, the forest in the highlighted cell (🔥) is burning. There are many possible ways that the fire can spread. Two possible scenarios are shown below.

0 hours

1	1	1	2
1	🔥	1	1
0	1	2	0
1	1	1	1

Scenario 1
3 neighbouring cells are burning.

2 hours

1	1	1	2
1	🔥	1	1
0	1	2	0
1	1	1	1

5 more neighbouring cells are burning.

4 hours

	1	1	2
1	🔥	1	1
0			0
1	1	1	1

Scenario 2
5 neighbouring cells are burning.

1	1	1	2
1	🔥	1	1
0	1	2	0
1	1	1	1

5 more neighbouring cells are burning.

1			2
	🔥		1
0	1		0
1	1	1	1

Strong wind conditions can affect the spread of fire into neighbouring cells.

N
W — E
S

= wind

Wind from the south-east

0 hours

2 hours

4 hours

Wind from the west

0 hours

2 hours

4 hours
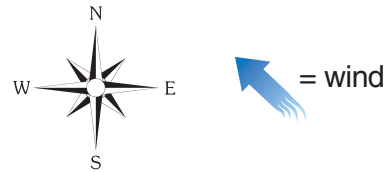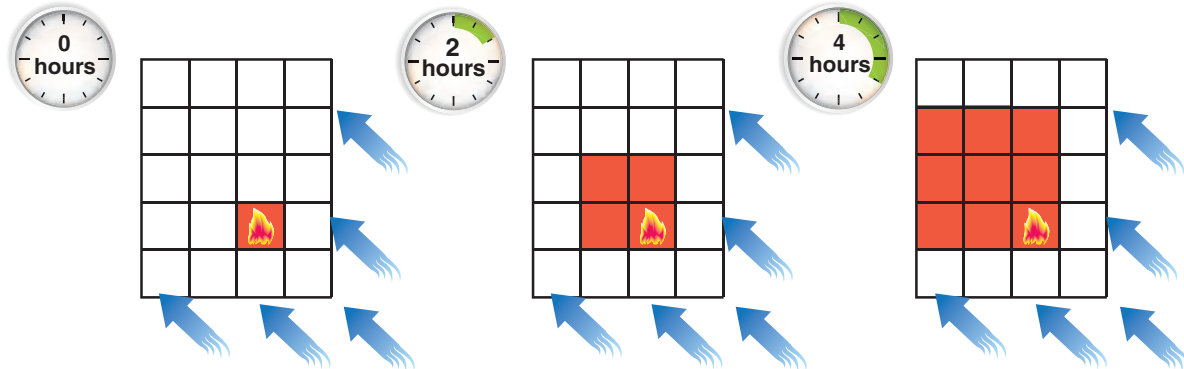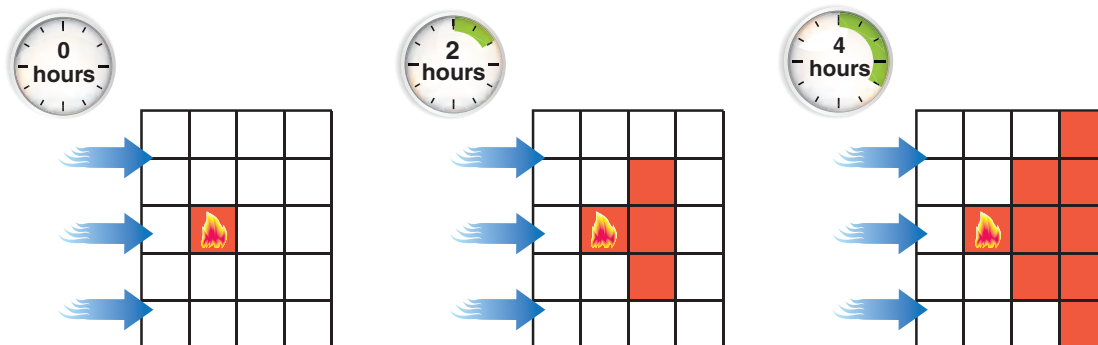
Wind **increases** the probability of fire spreading into neighbouring cells which are **downwind**, and wind **decreases** the probability of fire spreading into neighbouring cells which are **upwind**:

Fire-Spread Rating	Probability of Fire-Spreading		
	Neighbouring (no wind)	Downwind Neighbouring	Upwind Neighbouring
0	0	0	0
1	0.5	0.8	0.2
2	0.8	1.0	0.3

26. At time zero, the highlighted cell is burning and there is a strong, constant wind blowing from the south. There are people living in the areas within cell A and cell B.

   What is the minimum time it would take the fire to reach cells A and B?
   What is the likelihood of the fire spreading to cells A and B within that time?

   Explain and justify your solution.
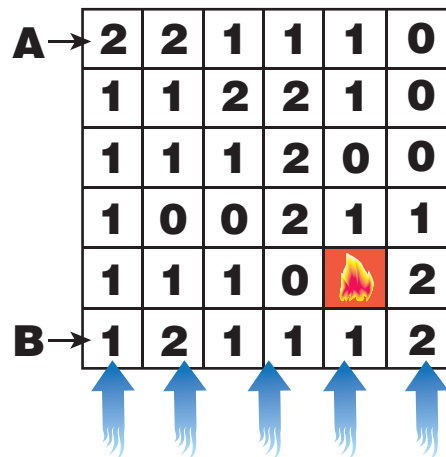
   This question is to be answered on paper.

Summary of Requirements for a Level 4:

▶ Provide time for fire to get to cell A and cell B (minimum 8 hours for both)
▶ Likelihood of the fire getting to cell A within that time (within 8 hours, it is high)
▶ Likelihood of the fire getting to cell B within that time (within 8 hours, it is very low)
▶ Provide evidence to justify solution
▶ Communicate solution in context

There are several possible solutions; a couple of solutions are given.

Solution 1

Cell A: The soonest the fire could reach cell A is 8 hours. As cell A is downwind from the highlighted cell, there is a high probability that the fire will spread to that cell in 8 hours (64%).

Probability = $1.0 \times 0.8 \times 0.8 \times 1.0 = 0.64$

Cell B: The soonest the fire could reach cell B is 8 hours, but because it is upwind, there is a low probability (0.24%) that the fire will spread to that cell.

Probability = $0.2 \times 0.2 \times 0.3 \times 0.2 = 0.0024$

Solution 2

The fastest the fire could reach both indicated cells is 8 hours. However, the fire is more likely to spread to cell A as it is downwind, and the most probable path is indicated by the grid below. The fire is not likely to spread to cell B because it is upwind. The most probable path is indicated by the grid below.
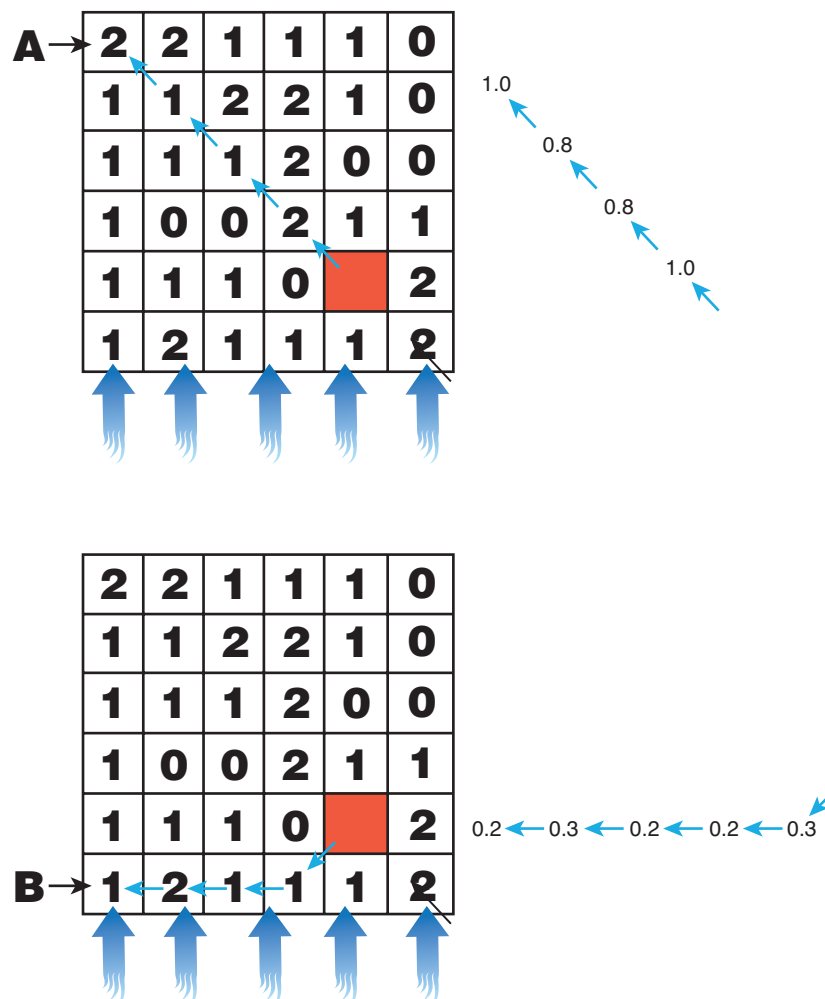
Forest Fires

Note:
There are 6 ways the fire could reach cell B in 8 hours, and the actual probability of spreading to cell B in that time is the sum of all of those cases *(P = 0.028)*. It is sufficient for students to provide one case and give that probability.

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0	🔥	2
B1	2	1	1	1	2

$$(0.2)^3(0.3) = 0.0024$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0	🔥	2
B1	2	1	1	1	2

$$(0.2)^4 = 0.0016$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0	🔥	2
B1	2	1	1	1	2

$$(0.2)^3(0.3) = 0.0024$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0	🔥	2
B1	2	1	1	1	2

$$(0.2)^4 = 0.0016$$

A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0	🔥	2
B1	2	1	1	1	2

$$(1.00)(0.2)^3 = 0.008$$

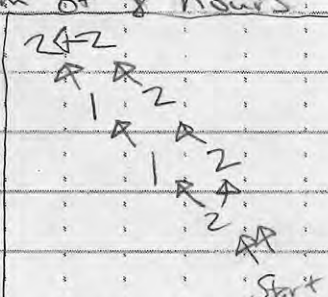| A2	2	1	1	1	0
1	1	2	2	1	0
1	1	1	2	0	0
1	0	0	2	1	1
1	1	1	0	🔥	2
B1	2	1	1	1	2

$$(1.00)(0.2)^2(0.3) = 0.012$$

Exemplar #1 – Score: 4

☑ The work demonstrates a proficient understanding of the situation.
- describes a likely path the fire could follow while spreading to both target cells
- identifies the correct probabilities based on the fire spread rating and the wind direction

☑ The strategy is effective and comprehensive.
- the probabilities for both (complete) paths are calculated
- total times for both paths are determined

☑ The logic references all aspects of the problem.
- overall probability is the product of the probabilities for each stage
- the total times are based on the number of stages within each path

☑ The reasoning is clear, detailed, and organized.
- the calculations for total time are described
- the probabilities for each stage are shown and correlate with the data table in the question
- the paths are drawn on a grid

for the people living in cell A they have a minimum of 8 hours before the fire arrives. Based on this path:
The odds of it reaching A in 8 hours is 64% based on this calculations: $(1.0 \cdot 0.8 \cdot 0.8 \cdot 1.0) \cdot 100\% = 64\%$
Should the fire not arrive in 8 hours it will have an 80% chance every 2 hours to arrive based on this path and this calculation: $(1.0 \cdot 1.0 \cdot 1.0 \cdot 0.8)$

As for B, the most direct path will arrive in a minimum of 8 hours as well, however the chance is only 0.24% based on the calculation: $(0.2 \cdot 0.2 \cdot 0.3 \cdot 0.2)$

Exemplar #2 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
- work is based on a likely path the fire could follow while spreading to both target cells – but the paths are not described
- identifies the correct probabilities based on the fire spread rating and wind direction

☑ The strategy is sensible but has some inconsistencies.
- the probabilities for both (complete) paths are calculated
- the total times are shown without support

☑ The logic references all aspects of the problem.
- overall probability is the product of the probabilities for each stage
- time for the complete path is shown

☑ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- the calculations for total time are not shown
- the probabilities for each stage are shown and correlate with the data table in the question
- the paths are not described or drawn on a grid
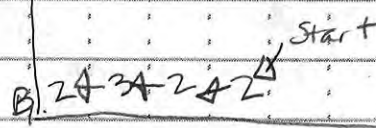
the minimum amount of time it would take is 8h for the
fire to reach A and B
    1 x 0.8 x 0.8 x 1
there is a 64% chance the fire will reach A in 8h
    0.2 x 0.2 x 0.3 x 0.3 x 0.2
there is a 0.24% chance the fire will reach B in 8h

Exemplar #3 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
- work is based on a likely path the fire could follow while spreading to both target cells
- identifies the correct probabilities based on the fire spread rating and wind direction

☑ The strategy is sensible but has some inconsistencies.
- the probabilities for both (complete) paths are calculated
- the total times are shown without support

☑ The logic references all aspects of the problem.
- overall probability is the product of the probabilities for each stage
- time for the complete path is shown

☑ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- the calculations for total time are not shown
- the probabilities for each stage are shown and correlate with the data table in the question
- the paths are drawn on a grid

A is ~~upwind~~ downwind
Fire could reach A in 8 hours
The probability is
$$(1.0)(0.8)(0.8)(1.0) = 0.64$$

B is ~~downwind~~ upwind
Fire could reach B in 8 hours.
The probability is
$$(0.2)(0.2)(0.3)(0.2) = 0.0024$$

The probability that the fire reaches A in 8 hours is 64%, which is fairly high.
The fire can reach B in 8 hours, but the probability is very low, 0.0024 which is not very likely.

Exemplar #4 – Score: 3

☑ The work demonstrates an adequate understanding of the situation.
- work is based on a likely path the fire could follow while spreading to both target cells
- applies some of the correct probabilities based on the fire spread rating and wind direction

☑ The strategy is sensible but has some inconsistencies.
- the overall probabilities are shown but not supported (one is incorrect)
- total times for both paths are determined

☑ The logic references all aspects of the problem.
- overall probability is the product of the probabilities for each stage (one is incorrect)
- the total times are based on the number of stages within each path

☑ The reasoning is evident; most supporting calculations are shown; some structure is used to organize the components of the solution.
- the total time is determined based on the sequence of drawings
- the calculation of the probabilities for each stage are not shown
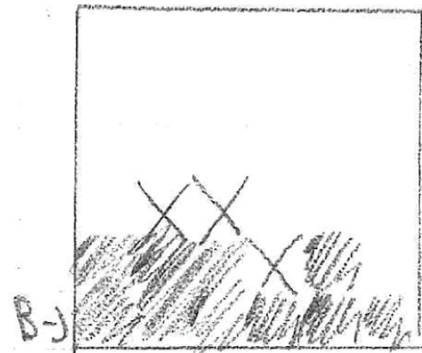- the paths are drawn on a grid

Location A   Location B

A→   B→
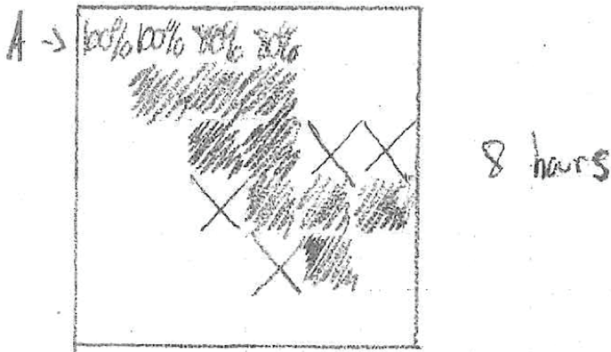
2 hours

100% 90% 80%   0%
0%   20% 20% 30%

A→   B→

4 hours

80% 100% 0% 0%
0%   80%
                              20%

continued on next page

Location A

A →

80% 100% 100%

6 hours

Location B

0% 0%
80%
B → 30%

A → 100% 100% 90% 80%

8 hours

B → 80%
20%

A →
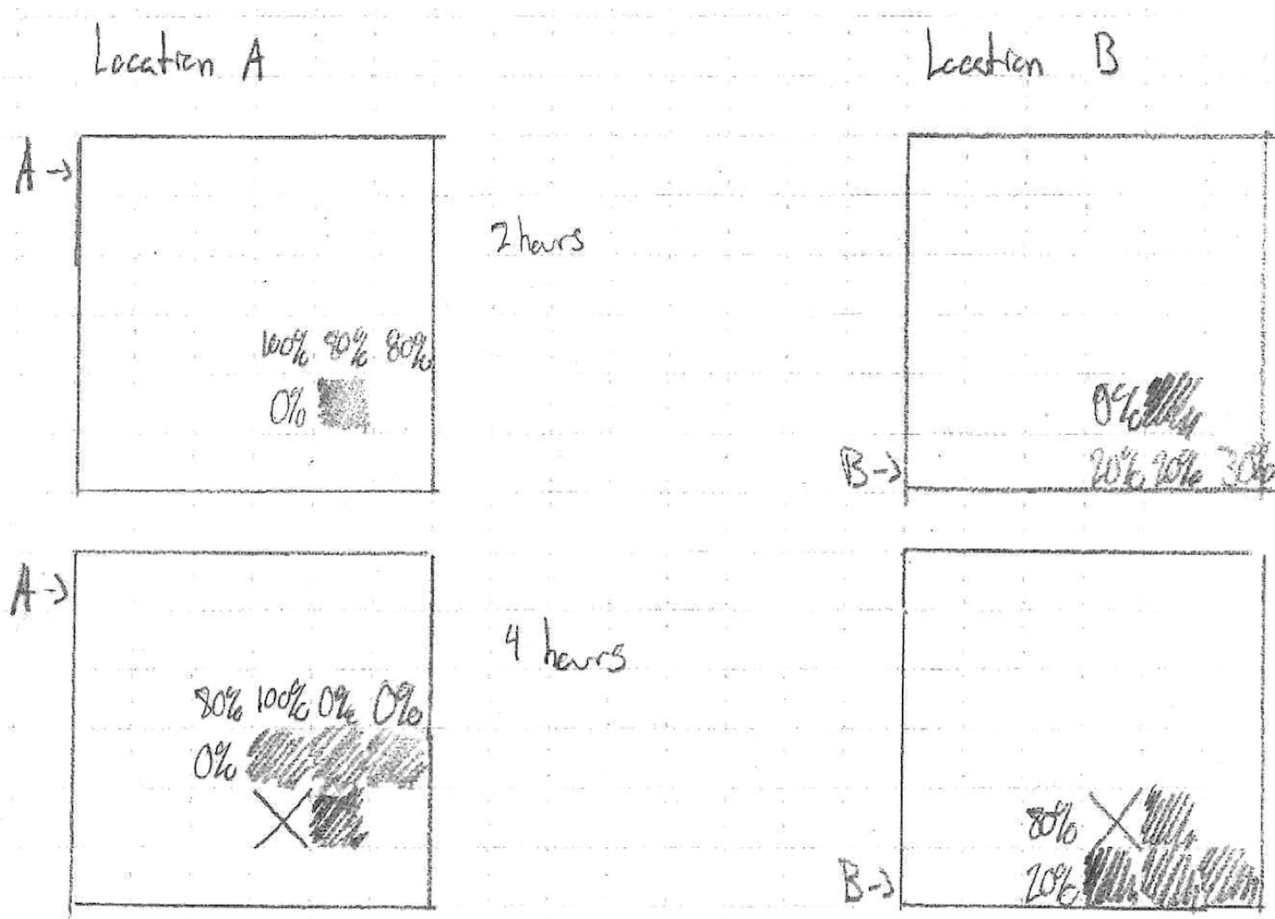
10 hours

B →
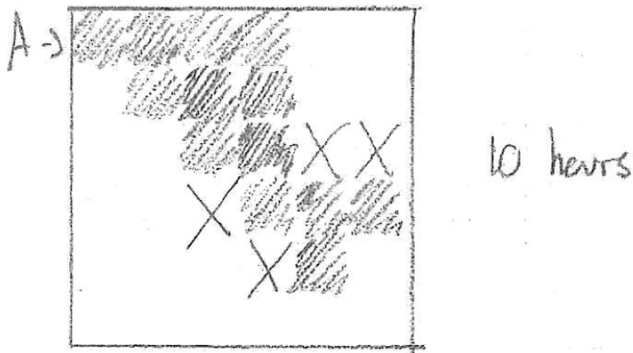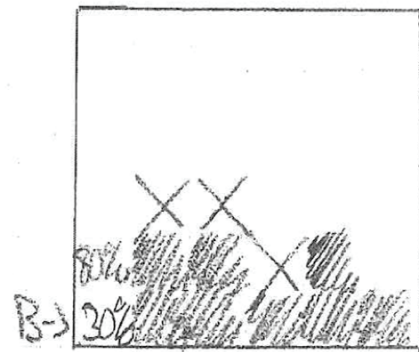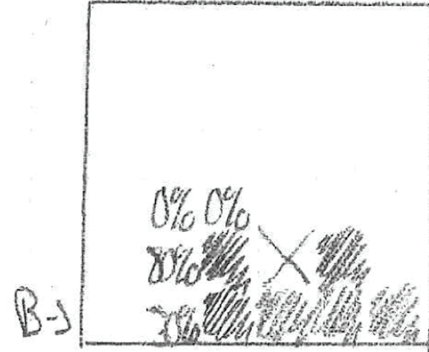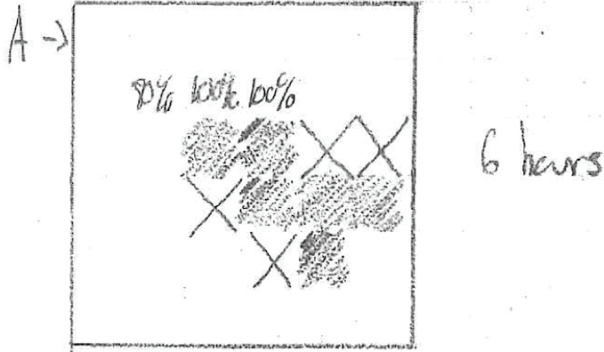
the fire will reach
location A in 10 hours.
The people should evacuate
ias the fire will arrive soon.
The probability is 100%

Worst case the fire
the fire will reach
location B in 10 hours,
however the probability is
0.24%. So the people are
probably safe

Exemplar #5 – Score: 2

☑ The work demonstrates a basic understanding of the situation.
- work is based on a likely path the fire could follow– but just to one of the target cells
- limited reference to the concept of probability

☑ The strategy is unclear and/or incomplete.
- overall probabilities are not determined for either path
- the total times are based on the number of stages, but for just one of the paths

☑ The logic correctly references some aspects of the problem.
- neither the single cell or total probabilities are addressed
- time for one complete path is presented; the other is not referenced

☑ The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
- the total time is determined based on the sequence of drawings
- the probability calculations are not shown
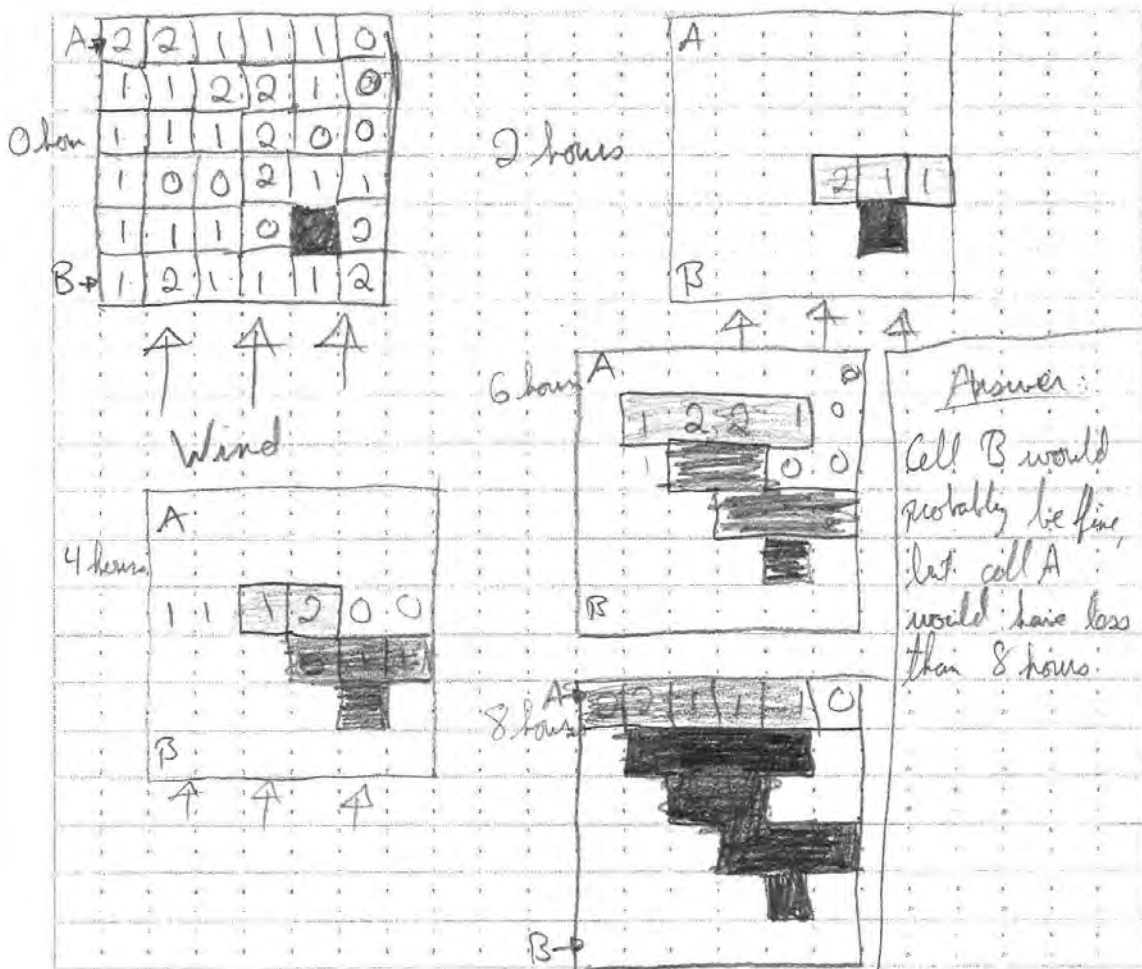- one of the paths is described using a sequence of grids

Exemplar #6 – Score: 2

✓ The work demonstrates a basic understanding of the situation.
- work is based on a likely path the fire could follow– but just to one of the target cells
- limited reference to the concept of probability

✓ The strategy is unclear and/or incomplete.
- the overall probabilities are shown but not supported (both are incorrect)
- the total times are inferred in the diagram, but the process is not clear

✓ The logic correctly references some aspects of the problem.
- time for one complete path is inferred; the time for the other path is not referenced
- the single cell probabilities are not shown or referenced

✓ The reasoning is unclear; supporting calculations are not well represented; the solution has some structure.
- the total time is not supported
- the probability calculations are not shown
- the paths are inferred on a grid

Exemplar #7 – Score: 2

☑ The work demonstrates a basic understanding of the situation.
- addresses the concept the total time for the fire to spread to the target cells
- references the concept of probability

☑ The strategy is unclear and/or incomplete.
- the probabilities are based on conjecture
- there is some explanation of the total times involved for the fire to spread

☑ The logic correctly references some aspects of the problem.
- fragmented logic determining a total time
- the single cell probabilities are referenced but not connected

☑ The reasoning is unclear; supporting calculations are not well represented; solution has limited structure.
- the probability calculations are not shown
- the paths are described using grids; the total time is represented on the drawings

Upwind = blank neighbourhoods / boxes => decreasing probability of fire due to wind

Downwind - shaded neighbourhoods / boxes/cells => increase probability of fire due to wind

2 hours => 8 cells shaded

4 hours -> 16 cells shaded

In cell B - they will have a long time as the wind is blowing and it's an upwind which means it's a decreasing probability of fire due to wind

Also, not even affected probably

4 hours ---- 16 cells shaded
x hours ---- 20 cells shaded

80 cells = 16x / $\frac{1}{16}$
x = 5 hours

Possibly, as mentioned, it's a strong, constant wind blowing.

It will take about 5 hours in order for Cell A to get affected due to fire.

Moreover cell A has a fire-speed rating of 2, meaning that it has an 80% / 0.8 probability of neighbouring prob fire spreading or downwind 1.0. On the other hand cell B, has a 0.5/50% probability of neighbouring fire spreading old d. 0.8 downwind.

Exemplar #8 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
- the time for the fire to spread is stated (incorrectly) but not supported
- probabilities are not considered

☑ The strategy is ineffective.
- a strategy is not evident

☑ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- the logic behind the solution is not shown

☑ The reasoning is missing or irrelevant; supporting calculations are not shown.

I would probaly take around 3-4 hours to reach B withethe wind coming from the west.

For A i think the least amount of time would be 4.5 hours

Exemplar #9 – Score: 1

☑ The work demonstrates an inadequate understanding of the situation.
- the time for the fire to spread is stated but not supported
- probabilities are restated

☑ The strategy is ineffective.
- the probabilities between single cells are restated, but the overall probability is not considered
- the total times are stated without any supporting rationale

☑ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- time for the complete path is presented without supporting logic
- total probabilities are not addressed

☑ The reasoning is missing or irrelevant; supporting calculations are not shown.

Exemplar #10 – Score: 1

✓ The work demonstrates an inadequate understanding of the situation.
- the time for the fire to spread is stated but not supported
- probabilities are restated

✓ The strategy is ineffective.
- the probabilities between single cells are restated, but the overall probability is not calculated
- the total times are stated without any supporting rationale

✓ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- time for the complete path is presented without supporting logic
- total probabilities calculations are incomplete

✓ The reasoning is missing or irrelevant; supporting calculations are not shown.

Minimum time fire to spread to A and B
A: The fire will reach point A in 5 hours
   $0.3 \times 0.2 =$

B: Point B is below the point where the fire starts and because of that and the upward wind, point B isn't affected.

Exemplar #11 – Score: 1

✓ The work demonstrates an inadequate understanding of the situation.
- the time for the fire to spread is shown but is incorrect
- the concept of probabilities are discussed in vague terms

✓ The strategy is ineffective.
- overall probability is presented but not supported
- the total times are based on the number of stages for each path

✓ The logic does not reference the problem. The solution may contain fundamental mathematical errors.
- time for the complete path is presented
- total probabilities are shown but not calculated

✓ The reasoning is missing or irrelevant; supporting calculations are not shown.
- the probabilities for each stage are shown and correlate with the data table in the question
- the paths are described; the total time is represented on the drawings

▨ 2h.
▧ 4h.
▤ 8h
⊠ 16h

In the worst case scenario,
* A cell people have 16 hours before they are in danger

* B cell people also have 16 hours before they are in danger.

Since the wind is pointing north towards Ⓐ cell, it is down wind, so Ⓐ cell has a higher probability of burning within the 16 hours the fire would take to spread in the worst case scenario.

People in cell Ⓑ could be more relaxed because its not very likely that the fire reaches cell Ⓑ in the 16 hours, for B cell is upwind.

Exemplar #12 – Score: 0

☑ The work is not relevant to solving the problem.

☑ Response does not address the purpose of the task.
- probability is not referenced
- a time is stated, but is not supported or linked to the context of the problem

☑ The solution is incorrect with no work shown.

☑ Information is simply recopied from the problem. A time value is stated with no reference or support.

it would take about 5 hours