Lesson 1 – Systems of Linear Equations and Graphs

The equation of a straight line graph is also called a linear Equation
A pair of linear equations is called a linear system.
A solution to a linear system is the coordinates of the point where the pair of lines intersects.

solution: (x, y)

How to find the solution to a linear system graphically?

1. Graph one line on a grid (y = mx + b).
2. Graph the second line on the same grid (y = mx + b).
3. The point where the two lines intersect is the answer!
   Write the answer as a coordinate (x, y). *Note:* Write *no solution* if no such point exists.

Example 1: Solve the linear system graphically.

① $x - y = -2$
$4x + 2y = 16$

$y = x + 2$
$m = 1 \rightarrow \frac{1}{1}$
y intercept (0, 2)

② $-4x$

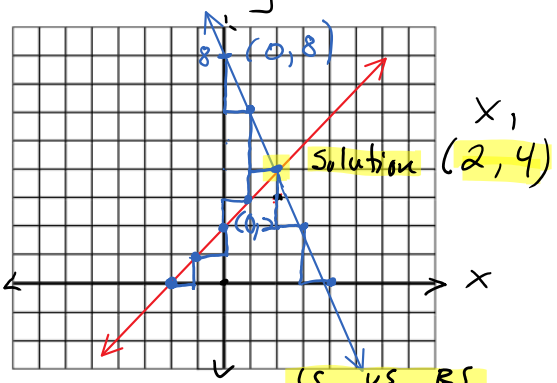
$\frac{2y}{2} = \frac{-4x + 16}{2}$
$y = -2x + 8$
$m = -2$
y intercept (0, 8)

How can you check if your answer is correct?
Verify Solution (2, 4)

① $x - y = -2$
LS ? RS
$2 - (4) = -2$
$-2 = -2$

② $4x + 2y = 16$
$4(2) + 2(4) \stackrel{?}{=} 16$
$8 + 8 = 16$

So LS = RS
16 = 16

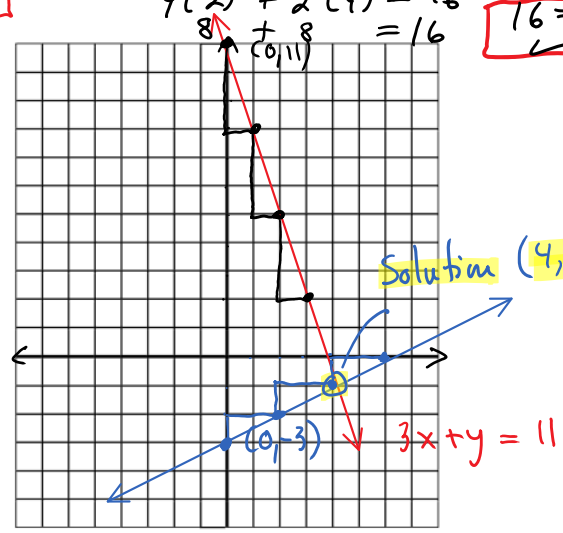


Example 2: Solve the linear system graphically.

① $y = mx + b$
$3x + y = 11$
$x - 2y = 6$

$y = -3x + 11$
$m = -3 \rightarrow \frac{-3}{1}$
y intercept (0, 11)

② $x - 2y = 6$
$+2y \quad +2y$
$x = 2y + 6$
$-6 \quad -6$
$\frac{2y}{2} = \frac{1x - 6}{2}$
$y = \frac{1}{2}x - 3$
$m = \frac{1}{2}$
y intercept (0, -3)



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Example 3: David earns $40 plus $10 per hour. Ranjit earns $50 plus $8 per hour.

(a) Represent the linear system relating the earnings numerically (table of values) and graphically.

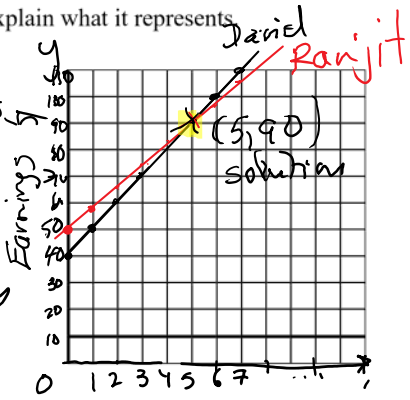
(b) Identify the solution to the linear system and explain what it represents.

let x = # of hours

David / Ranjit → y = Earnings

x	y	
0	40	50
1	50	58
2	60	66
3	70	74
4	80	82
5	90	90
6	100	98
7 (hrs)	110	106

↑ The solution to the system



David

$y = 10x + 40$

Ranjit

$y = 8x + 50$

$y = 8(2) + 50$

$= 16 + 50$

$= 66$

b) a) 5 hours both David & Ranjit earn $90. (5, 90)

Example 4: For the system of linear equations, verify whether the given point (-3,-2) is a solution. Explain what the results would show on a graph.

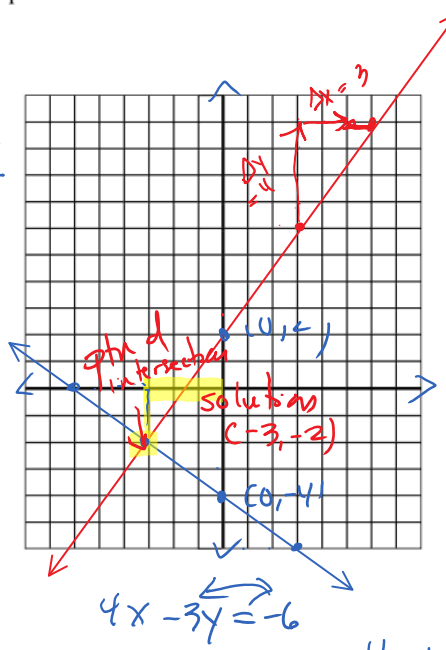
$2x + 3y = -12$

$4x - 3y = -6$

(-3, -2)

$2x + 3y = -12$			$4x - 3y = -6$		
LS	vs	R	LS	vs	RS
$2(-3) + 3(-2)$		-12	$4(-3) - 3(-2)$		-6
$-6 - 6$			$-12 + 6$		
-12	=	-12	$-$		-6
	RS		LS	≠	RS

Yes (-3, -2) is a solution!



$2x + 3y = -12$

$\frac{3y}{3} = \frac{-2x}{3} - \frac{12}{3}$

$y = -\frac{2}{3}x - 4$

$m = -\frac{2}{3}$

y int. (0, -4)

$\frac{3y}{3} = \frac{4x}{3} + \frac{6}{3}$

$y = \frac{4}{3}x + 2$

$m = \frac{4}{3}$

y int. (0, 2)

Homework: ~~Worksheet "What were the Headlines…"~~ + Pg. 129.

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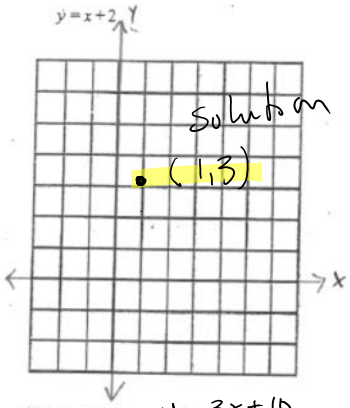
Section 8.1 Extra Practice

Key

1. Solve each system of linear equations graphically.

a) $y = -2x + 5$

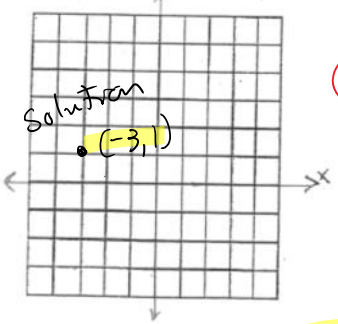
$y = x + 2$



Solution (1, 3)

b) $3x - y = -10$ $y = 3x + 10$

$2x + y = -5$ $y = -2x - 5$

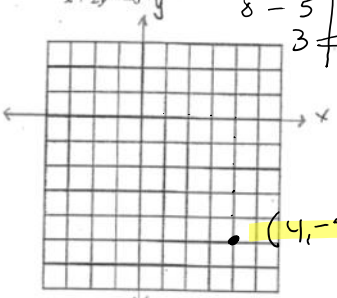


Solution (−3, 1)

2. Solve the system of linear equations graphically. Then, verify your solution.

a) $2x + y = 3$ ①

$x + 2y = -6$ ②



(4, −5)

verify

	LS	RS
①	$2(4) - 5$	3
	$8 - 5$	
	$3 \neq 3$ ✓	

② $x + 2y = -6$

3. Is the given point a solution to the system of linear equations?

a) $y = 5x + 13$, $y = -7x - 35$ (4, 7) No.

b) $4x - 5y = 20$, $x + 3y = -29$ (−5, −8) Yes.

LS ≠ RS

Show verification steps for each eqn & point. Does left side = right side?

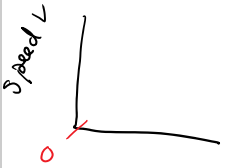
4. **Extension**

With a tailwind, a particular airplane travels at 420 km/h. When flying against the wind, the airplane can travel at only 310 km/h. Let the airplane's speed in still air be v kilometres per hour. Let the wind speed be w kilometres per hour. The system of linear equations $v + w = 420$ and $v - w = 310$ represents this information. Solve the linear system graphically to determine the wind speed and the speed of the plane in still air.

Challenging to graph!

(w, v) approx.

Solution: (55, 365)



-1 (0, 420)

$v = -w + 420$

$v = w + 310$

(1, 310)

Draw graph with care.

- Can solve with table of values

w	v_1	v_2
0		

Graph of lines ↓ too.

$m = (\ )$

$m = (\ )$

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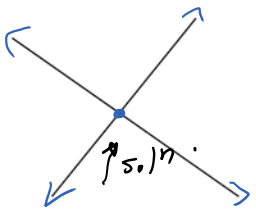
video 2:

Lesson 4 – Number of Solutions of a Linear System

There are 3 possibilities when determining the number of solutions of a linear system:

1. Intersecting Lines



Number of Solutions: 1

This system is called consistent.

Example: ① $5x + 6y = 1$
② $6x + 2y = -3$

Put each equation in y = mx + b form:

① $5x + 6y = 1$

$\frac{6y}{6} = \frac{-5x}{6} + \frac{1}{6}$

$y = \frac{-5}{6}x + \frac{1}{6}$

$m = -\frac{5}{6}$

② $6x + 2y = -3$

$-6x \quad -6x$

$\frac{2y}{2} = \frac{-6x}{2} - \frac{3}{2}$

$y = -3x - \frac{3}{2}$

$m = -3$

y are solution only. 1
then slopes must be different
y inter. can be identical or different

2. Parallel Lines

Number if Solutions: 0 b/c no solution

This system is called inconsistent.

Example: $2x + y = 4$ and $2x + y = 8$ or $2x + y = 4$ and $4x + 2y = 3$

* Parallel lines Have the same slope but dif. y intercepts.

Put each equation in y = mx + b form:

$y = -2x + 4$

$y = -2x + 8$

same slope = parallel
but dif y intercept

$y = -2x + 4$

$\frac{2y}{2} = \frac{-4x}{2} + \frac{3}{2}$

$y = -2x + \frac{3}{2}$

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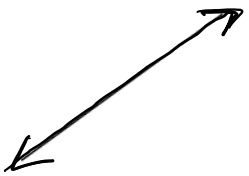
3. Coincident Lines

Number of solutions: infinite.

This system is consistent.

Example: ① $2x + y = 4$
② $4x + 2y = 8$



Put each equation in $y = mx + b$ form:

① $y = -2x + 4$

② $4x + 2y = 8$
$-4x \quad -4x$
$\frac{2y}{2} = \frac{-4x}{2} + \frac{8}{2}$
$y = -2x + 4$

same "m" & y intercept so same line.

Example 1: Given $x - 2y = 8$. Write a second equation to form a linear system with:

(a) infinitely many solutions (b) no solution (c) only one solution

same slope & y intercept.

same equation → unsimplified

same slope → but y inter differs

Solution:
(a) $x - 2y = 8$ → $\frac{2y}{2} = \frac{x}{2} - \frac{8}{2}$, $y = \frac{x}{2} - 4$
$2(x - 2y = 8)$
$2x - 4y = 16$ simplified
$\frac{4y}{4} = \frac{2x}{4} - \frac{16}{4}$, $y = \frac{1}{2}x - 4$

(b) $x - 2y = 8$ → $y = \frac{x}{2} - 4$
$y = \frac{x}{2} + 7$
same slope, y inter differ, so parallel.

(c) $x - 2y = 8$ → $y = \frac{x}{2} - 4$
dif slope!
$y = -2x - 4$
$y = \frac{x}{3} + 2$
dif slope! intercept does not matter.

Example 2: Is (1, -2) a solution of the following system?

$3x - y = 1$ → $y = 3x - 1$
$x + y = 3$ → $y = -x + 3$
consistent one solution!

- Check to see if the system is consistent, otherwise if inconsistent the system is a pair of parallel lines, and there is no solution.
- This system is: INCONSISTENT / CONSISTENT

(1, -2)

LS vs RS
$3x - y \stackrel{?}{=} 1$
$3(1) - (-2) \stackrel{?}{=} 1$
$3 + 2 \stackrel{?}{=} 1$
$5 \neq 1$
pt (1, -2) not a solution

$x + y = 3$
$1 \quad -2$
$-1 \neq 3$

Assignment: Pg 135

Online textbook!
pg. 404/405
1, 2, 3, 6, 7